MCS21 – Calculus
Exam 3 Review Sheet

Topics:

- the definition of continuity
- the definition of the derivative
- the alternate definition of the derivative
- the power rule
- non-differentiability of functions (corner/cusp, points of discontinuity, vertical tangent)
- differentiability of piecewise-defined functions

Practice:

1. Find the derivative of each function.

   (a) $y = 3x + 4x^2$ (b) $f(x) = 5\pi^2 x - 8x^4$ (c) $h(x) = -2x^5 + 5\sqrt{x} - \sqrt[3]{x}$

2. Use the <u>definition of the derivative</u> to find $f'(x)$ if

   (a) $f(x) = \sqrt{x+2}$ (b) $f(x) = x^2 - 5x + 1$

3. Use the <u>*alternate* definition of the derivative</u> to find $f'(1)$ if $f(x) = \dfrac{7}{x-4}$.

4. Given $f(x) = \begin{cases} x^2 - 1 & x \neq 1 \\ 4 & x = 1 \end{cases}$. Which of the following are true? Explain.

   (i) $\lim_{x \to 1} f(x)$ exists (ii) $f(1)$ exists (ii) f is continuous at $x = 1$

5. Find $\dfrac{dx}{d\theta}$ if $x = 2\theta^{-2} - \theta$

6. If $f'(a)$ does not exist, which of the following *must* be true?

   (A) $f(x)$ is discontinuous at $x = a$.
   (B) $\lim_{x \to a} f(x)$ does not exist.
   (C) f has a vertical tangent at $x = a$.
   (D) f has a "hole" at $x = a$.
   (E) None of the above must be true.

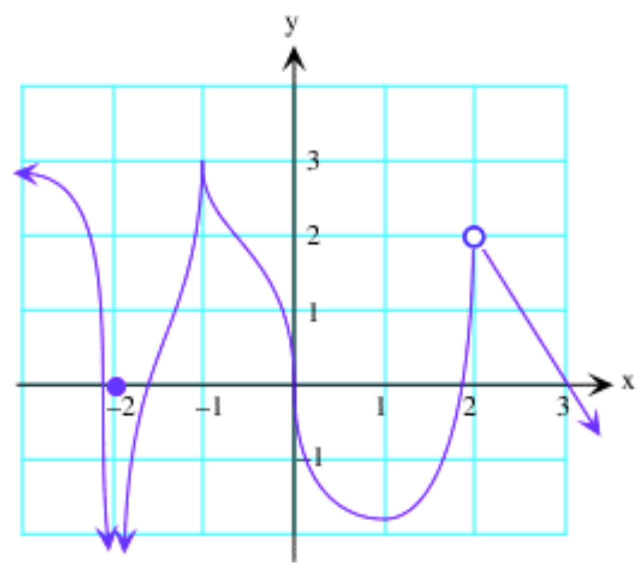
7. Which statement is true for the function $f(x)$, if $f(x) = \begin{cases} x+1, & \text{if } x \le 1 \\ x^2+1, & \text{if } x > 1 \end{cases}$?

   (A) $f(x)$ is continuous and differentiable at $x = 1$.
   (B) $f(x)$ is continuous but non-differentiable at $x = 1$.
   (C) $f(x)$ is not continuous but is differentiable at $x = 1$.
   (D) $f(x)$ is not continuous and also non-differentiable at $x = 1$
   (E) $f(x)$ has a removable discontinuity and is differentiable at $x = 1$.

8. Let $f(x)=\begin{cases} x^3+16 & x<\frac{1}{2} \\ \frac{3}{4}x^2 & x\ge\frac{1}{2} \end{cases}$. Determine whether f is differentiable at $x=\frac{1}{2}$. If so, find the value of the derivative there.

9. Let $f(x)=\begin{cases} x^2 & x\le 2 \\ mx+b & x>2 \end{cases}$. Find values of m and b that make f differentiable everywhere.

10. Use accompanying graph of $f(x)$ to answer parts (a) through (h) below.



(a) $\lim_{x\to -2} f(x) =$ ________

(b) $f'(3) =$ ________

(c) $f(2) =$ ________

(d) $f(-2) =$ ________

(e) Create a true statement by filling in the box with one of the three symbols: >, <, or = .

$f'\left(-\frac{1}{2}\right)$ ☐ $f'\left(\frac{3}{2}\right)$

(f) State all values of x where $f(x)$ is discontinuous.

(g) State all values of x where $f(x)$ is not differentiable.

(h) The value of $f'(x)$ is zero when:

(A) $x=-1$ (B) $x=0$ (C) $x=1$ (D) $x>2$ (E) $x<-1$

11. Let $f(x)=\begin{cases} 3bx^3-2 & x\ge 1 \\ x^2-ax^4 & x<1 \end{cases}$. Find the values of a and b such that $f(x)$ is differentiable at $x=1$.

12. Find $\frac{dy}{dx}$ in simplest form.

(a) $y=\frac{-3}{x^6}-2x^{\frac{11}{7}}$ (b) $y=\sqrt[5]{x^3}+\pi x-2$ (c) $y=kx^4-7cx^3+11d^2$

(c, k, and d are constants)