Name: _____ **Precalculus Unit 4, Lesson 9**

Aim: How do we test for upper and lower bounds for roots?

I. Do Now:

1. Given $f(x) = x^4 - x^3 - x^2 - 5x - 30$
   (a) Find all possible rational zeros.
   (b) Use synthetic division and your answer to part (a) to find all real roots of $f(x)$.

II. Upper and Lower Bounds of Roots (Another "trick" to add to our root-finding arsenal.)

Given $f(x)$, a polynomial function with real coefficients and a positive leading coefficient.

If $f(x)$ is divided by $(x - c)$ using synthetic division,

- If $c > 0$ and every number in the bottom row is either positive or zero, then *c* is an *upper bound* for the real zeros of $f(x)$ (i.e., no real roots can be greater than *c*)
- If $c < 0$ and the numbers in the bottom row are alternatively positive and negative (zero counts as either positive or negative), then *c* is a *lower bound* for the real zeros of $f(x)$ (i.e., no real roots can be less than *c*)

III. Illustration

Consider the function $f(x) = 6x^3 - 4x^2 + 3x - 2$, with possible rational zeros $\pm 1, \pm 2, \pm \frac{1}{2}, \pm \frac{1}{2}, \pm \frac{1}{3}, \pm \frac{2}{3}, \pm \frac{1}{6}$.

Use synthetic division on the following numbers: 1, -1, $\frac{2}{3}$, and $\frac{1}{2}$

IV. Applications

2. Given $g(x) = x^3 - 21x^2 + 18x + 40$.
   (a) Find all possible rational zeros.
   (b) Use synthetic division to find all the real roots.

3. Use synthetic division to test if the given value is an upper bound, a lower bound, or neither.
   Given: $f(x) = x^3 - 4x^2 + x + 6$.
   Test $x = -2$ and $x = 4$.

HW32
- p. 110: 73, 98
- p. 124: 49, 59, 70d, 71 –74 (just verify upper and lower bounds; do not find real zeros)