Name: _____ **Precalculus Unit 4, Lesson 13**

Aim: How do we factor polynomials completely over the complex numbers?

I. Do Now:

1. Find all real roots of $f(x) = x^4 - 5x^3 + 3x^2 + x$.

II. Every polynomial of degree $n > 0$, with real coefficients, can be written as the product of linear and quadratic factors with real coefficients, where the quadratic factors have no real zeros. (A quadratic factor with no real zeros is said to be *irreducible* over the reals.)

2. Express the polynomial function $f(x) = x^4 - x^2 - 20$
   (a) as the product of factors that are irreducible over the *rationals*.
   (b) as the product of factors that are irreducible over the *reals*.
   (c) in completely factored form (over the *complex numbers*)

III. Applications

3. Express the polynomial function $f(x) = x^4 + x^2 - 6$
   (a) as the product of factors that are irreducible over the *rationals*.
   (b) as the product of factors that are irreducible over the *reals*.
   (c) in completely factored form (over the *complex numbers*)

4. Find all roots of $f(x) = x^4 - 2x^3 - 6x^2 + 22x - 15$, if one root is $2 - i$.

5. For each polynomial
   (i) completely factor it over the real numbers
   (ii) completely factor it over the complex numbers
   (iii) find all zeros

   (a) $f(x) = x^4 + x^3 + 7x^2 + 9x - 18$

   (b) $f(x) = 8x^4 + 50x^3 + 43x^2 + 2x - 4$

   (c) $f(x) = x^5 - x^4 + 7x^3 - 7x^2 + 12x - 12$