

Aim: What is an infinite geometric series?**I. Do Now:**

1. Evaluate each sum:

(a) $\sum_{n=1}^{40} 4n - 17$

(b) $\sum_{n=1}^5 2^n$

(c) $\sum_{n=1}^5 \left(\frac{1}{2}\right)^n$

2. Given each series, calculate the first five partial sums.

(i) $\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \dots$

(ii) $\frac{1}{2} + 1 + 2 + 4 + \dots$

$S_1 =$

$S_1 =$

$S_2 =$

$S_2 =$

$S_3 =$

$S_3 =$

$S_4 =$

$S_4 =$

$S_5 =$

$S_5 =$

II. Definitions:

Since the *sequence of partial sums* approaches _____, then we say that the n th partial sum or the infinite series _____.

If the sequence of partial sums does not approach a number, then we say that the infinite series _____.

III. Formula: Two ways to derive the formula:(i) Consider the formula for the sum of the first n terms of a geometric sequence:

$$S_n = \frac{a_1(1-r^n)}{1-r}$$

(ii) Let $S = a_1 + a_1r + a_1r^2 + a_1r^3 + \dots$

Then, $S = a_1 + r(a_1 + a_1r + a_1r^2 + \dots)$

Sum of an Infinite Geometric Series:

$$S_\infty = \quad , \text{ when } -1 < r < 1$$

IV. Applications:

3. Determine whether the infinite geometric series converges or diverges.

If it converges, determine its sum.

(a) $\frac{1}{2} - \frac{1}{4} + \frac{1}{8} - \frac{1}{16} + \dots$

(b) $27 - 9 + 3 - 1 + \dots$

(c) $4 - 8 + 16 + -32 + \dots$

4. Use an infinite geometric series to express each repeating decimal as a fraction.

(a) 0.2222... (b) 0.252525...

5. If the first term of an infinite geometric series is 21 and its sum is 63, find the common ratio.

6. If the sum of an infinite geometric series is $\frac{24}{7}$ and the common ratio is $\frac{3}{4}$, find the first term.

7. $\sum_{n=1}^{\infty} 4(-0.6)^{n-1}$