Name: ______________________ **Precalculus Unit 12, Lesson 3**

Aim: What are some more sophisticated techniques for computing limits?

I. Do Now:

1. Find the limit, if it exists.

(a) $\lim_{x\to 5} \pi$ (b) $\lim_{x\to 3} \frac{x^2-9}{x^2+x-12}$ (c) $\lim_{x\to 4} \frac{\sqrt{x}-2}{x-4}$

2. Find the equation of the horizontal or slant asymptote of each rational function.
(Remember to consider the degree of the numerator and denominator.)

(a) $f(x)=\frac{3x}{x^2+1}$ (b) $g(x)=\frac{3x^2}{x^2+1}$ (c) $h(x)=\frac{3x^3}{x^2+1}$

II. Recall:

(1) If degree of numerator < degree of denominator, then: ______________________________

(2) If degree of numerator = degree of denominator, then: ______________________________

(3) If degree of numerator > degree of denominator, then: ______________________________

III. Development: *Computational Techniques for Finding Limits*

(1) Direct Substitution (If the result is $\frac{0}{0}$ or $\frac{\infty}{\infty}$, use another method.)

(2) Factor and cancel (you may need to use synthetic division)

(3) Rationalize the numerator or denominator

(4) Simplify the complex fraction.

(5) If $f(x)$ is rational function $f(x)=\frac{g(x)}{h(x)}$, $h(x)\neq 0$.

To find $\lim_{x\to\infty} f(x)$ or $\lim_{x\to-\infty} f(x)$, use the rules for horizontal asymptotes.

IV. Applications: Find the limits, if they exist. If they do not exist, write one of the following DNE, DNE $(+\infty)$, or DNE $(-\infty)$.

3. $\lim_{x\to\infty} \frac{2x^2-4x+3}{3x^2-1}$

4. $\lim_{x\to\infty} \frac{x^2-4}{x^3-10}$

5. $\lim_{x\to\infty} \frac{x^3}{x^2+x}$

V. Mixed Practice:

6. $\lim_{x\to 0} \frac{\frac{1}{2-x}-\frac{1}{2}}{x}$

7. $\lim_{x\to 1} \frac{x^3+5x^2+2x-8}{x^3+3x^2-x-3}$

8. $\lim_{x\to 3} \frac{x^3-27}{x^3-5x^2-2x+24}$

(if time:)

9. Given $f(x)=3x-1$

Find $\lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$