Name: _____ **Precalculus Unit 6, Lesson 12**

Aim: How do we use matrices to send and decode secret messages?

I. Do Now:

1. Use a determinant to find the area of the triangle whose vertices have coordinates $(-3, 4), (-1, -2),$ and $(1, 3)$.

2. Use a determinant to find the area of the parallelogram whose vertices have coordinates $(-3, 4), (-1, -2), (1, 3),$ and $(3, -3)$.

3. Solve by Cramer's Rule
$3x - 4y = 8$
$2x + 2y = 6$

II. Cryptography

A *cryptogram* is a message written according to a secret code.
Matrix multiplication can be used to *encode* and *decode* messages.

To begin, let's assign a number to each letter of the alphabet:

0 = __ (blank)	7 = G	14 = N	21 = U
1 = A	8 = H	15 = O	22 = V
2 = B	9 = I	16 = P	23 = W
3 = C	10 = J	17 = Q	24 = X
4 = D	11 = K	18 = R	25 = Y
5 = E	12 = L	19 = S	26 = Z
6 = F	13 = M	20 = T	

Before encoding the message, we form *uncoded row matrices* of order $1 \times n$ for the message:

1. Convert the letters to numbers.
2. Partition the numbers into uncoded row matrices, each having n entries (in this case, $n =$ ___)

Example: "I WILL SEE YOU SOON"

$$\begin{bmatrix} 9 & 0 & 23 \end{bmatrix}\begin{bmatrix} 9 & 12 & 12 \end{bmatrix}\begin{bmatrix} \quad & \quad & \quad \end{bmatrix}\begin{bmatrix} \quad & \quad & \quad \end{bmatrix}\begin{bmatrix} \quad & \quad & \quad \end{bmatrix}\begin{bmatrix} \quad & \quad & \quad \end{bmatrix}$$

I W I L L

III. To encode a message:

1. Choose an $n \times n$ invertible matrix B, called the _____.
2. Multiply the uncoded row matrices by B to get the *coded row matrices*.

Example: $\begin{bmatrix} 9 & 0 & 23 \end{bmatrix}\begin{bmatrix} 1 & -2 & 2 \\ -1 & 1 & 3 \\ 1 & -1 & -4 \end{bmatrix} = \begin{bmatrix} \quad & \quad & \quad \end{bmatrix}$

B

Therefore, $\begin{bmatrix} \quad & \quad & \quad \end{bmatrix} \times \begin{bmatrix} 1 & -2 & 2 \\ -1 & 1 & 3 \\ 1 & -1 & -4 \end{bmatrix} = \begin{bmatrix} \quad & \quad & \quad \end{bmatrix}$

B

A(uncoded) AB(coded)

IV. To decode a message:

1. Find B^{-1}, called the _____.
2. Multiply _____ by _____.

Example:

$$\underset{AB}{\begin{bmatrix} & \\ & \end{bmatrix}} \times \underset{B^{-1}}{\begin{bmatrix} & \\ & \end{bmatrix}} = \underset{A\text{(uncoded)}}{\begin{bmatrix} & \\ & \end{bmatrix}}$$

V. Application:

4. Given the following coded message and encryption matrix B, decode the message.

$$\begin{bmatrix} 48 & -23 & -20 \\ 98 & -46 & -39 \\ 83 & -31 & -39 \\ 53 & -23 & -26 \\ 26 & -1 & -25 \end{bmatrix} \qquad B = \begin{bmatrix} 1 & -1 & 0 \\ 1 & 0 & -1 \\ 6 & -2 & -3 \end{bmatrix}$$