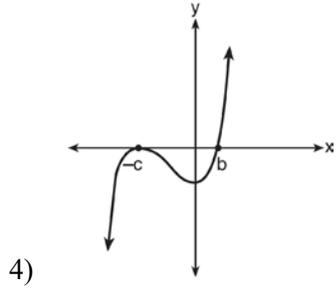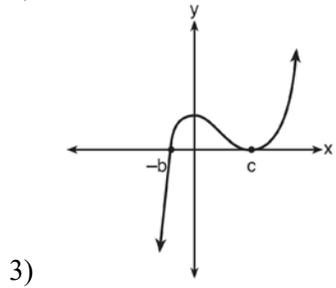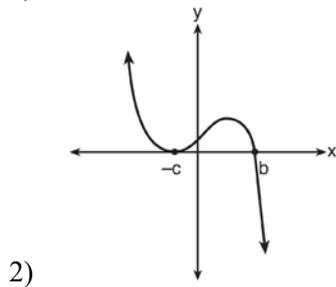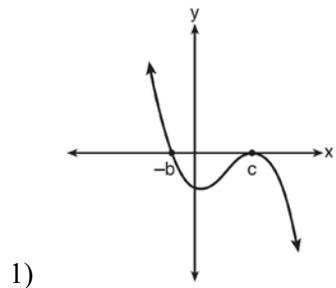## 2015 Algebra II Common Core State Standards Sample Items

1 If a, b, and c are all positive real numbers, which graph could represent the sketch of the graph of $p(x) = -a(x + b)\left(x^2 - 2cx + c^2\right)$?

1)

2)

3)

4)

2 Which equation represents a parabola with a focus of $(0,4)$ and a directrix of $y = 2$?

1) $y = x^2 + 3$

2) $y = -x^2 + 1$

3) $y = \dfrac{x^2}{2} + 3$

4) $y = \dfrac{x^2}{4} + 3$

3 If the terminal side of angle θ, in standard position, passes through point $(-4,3)$, what is the numerical value of $\sin \theta$?

1) $\dfrac{3}{5}$

2) $\dfrac{4}{5}$

3) $-\dfrac{3}{5}$

4) $-\dfrac{4}{5}$

4 A study of the annual population of the red-winged blackbird in Ft. Mill, South Carolina, shows the population, $B(t)$, can be represented by the function $B(t) = 750(1.16)^t$, where the t represents the number of years since the study began. In terms of the monthly rate of growth, the population of red-winged blackbirds can be best approximated by the function

1) $B(t) = 750(1.012)^t$

2) $B(t) = 750(1.012)^{12t}$

3) $B(t) = 750(1.16)^{12t}$

4) $B(t) = 750(1.16)^{\frac{t}{12}}$

5 Use the properties of rational exponents to determine the value of y for the equation:

$$\frac{\sqrt[3]{x^8}}{\left(x^4\right)^{\frac{1}{3}}} = x^y, \; x > 1$$

6   Write $(5 + 2yi)(4 - 3i) - (5 - 2yi)(4 - 3i)$ in $a + bi$ form, where y is a real number.

7   Use an appropriate procedure to show that $x - 4$ is a factor of the function $f(x) = 2x^3 - 5x^2 - 11x - 4$. Explain your answer.

8   Solve algebraically for all values of x:
$$\sqrt{x - 5} + x = 7$$

9   Monthly mortgage payments can be found using the formula below:

$$M = \frac{P\left(\frac{r}{12}\right)\left(1 + \frac{r}{12}\right)^n}{\left(1 + \frac{r}{12}\right)^n - 1}$$

M = monthly payment
P = amount borrowed
r = annual interest rate
n = number of monthly payments

The Banks family would like to borrow $120,000 to purchase a home. They qualified for an annual interest rate of 4.8%. Algebraically determine the *fewest* number of whole years the Banks family would need to include in the mortgage agreement in order to have a monthly payment of no more than $720.

10 Solve the following system of equations algebraically for all values of x, y, and z:
$$x + 3y + 5z = 45$$
$$6x - 3y + 2z = -10$$
$$-2x + 3y + 8z = 72$$

11 Write an explicit formula for a_n, the nth term of the recursively defined sequence below.
$$a_1 = x + 1$$
$$a_n = x(a_{n-1})$$
For what values of x would $a_n = 0$ when $n > 1$?

12 Stephen's Beverage Company is considering whether to produce a new brand of cola. The company will launch the product if at least 25% of cola drinkers will buy the product. Fifty cola drinkers are randomly selected to take a blind taste-test of products A, B, and the new product. Nine out of fifty participants preferred Stephen's new cola to products A and B. The company then devised a simulation based on the requirement that 25% of cola drinkers will buy the product. Each dot in the graph shown below represents the proportion of people who preferred Stephen's new product, each of sample size 50, simulated 100 times.
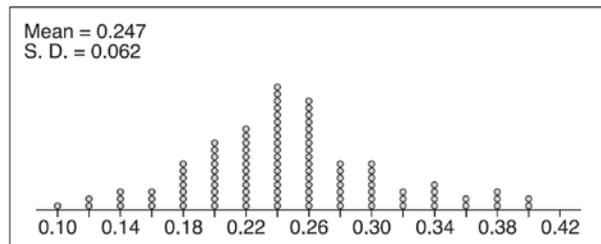

Proportion Preferring Stephen's Product

Assume the set of data is approximately normal and the company wants to be 95% confident of its results. Does the sample proportion obtained from the blind taste-test, nine out of fifty, fall within the margin of error developed from the simulation? Justify your answer. The company decides to continue developing the product even though only nine out of fifty participants preferred its brand of cola in the taste-test. Describe how the simulation data could be used to support this decision.

2

13 In contract negotiations between a local government agency and its workers, it is estimated that there is a 50% chance that an agreement will be reached on the salaries of the workers. It is estimated that there is a 70% chance that there will be an agreement on the insurance benefits. There is a 20% chance that no agreement will be reached on either issue. Find the probability that an agreement will be reached on *both* issues. Based on this answer, determine whether the agreement on salaries and the agreement on insurance are independent events. Justify your answer.

14 The ocean tides near Carter Beach follow a repeating pattern over time, with the amount of time between each low and high tide remaining relatively constant. On a certain day, low tide occurred at 8:30 a.m. and high tide occurred at 3:00 p.m. At high tide, the water level was 12 inches above the average local sea level; at low tide it was 12 inches below the average local sea level. Assume that high tide and low tide are the maximum and minimum water levels each day, respectively. Write a cosine function of the form $f(t) = A\cos(Bt)$, where A and B are real numbers, that models the water level, $f(t)$, in inches above or below the average Carter Beach sea level, as a function of the time measured in t hours since 8:30 a.m. On the grid below, graph one cycle of this function.
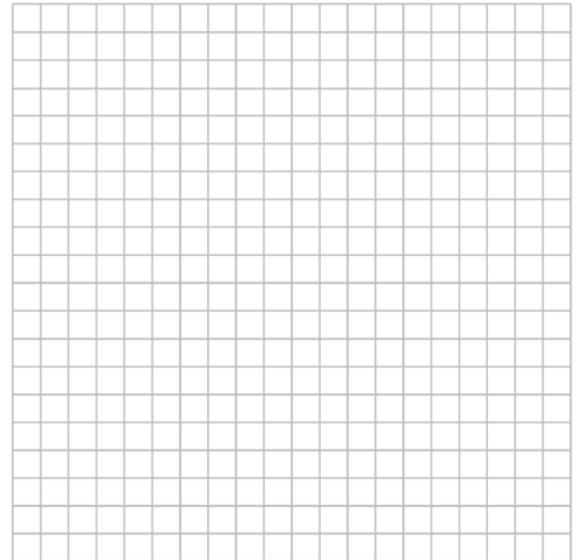
People who fish in Carter Beach know that a certain species of fish is most plentiful when the water level is increasing. Explain whether you would recommend fishing for this species at 7:30 p.m. or 10:30 p.m. using evidence from the given context.
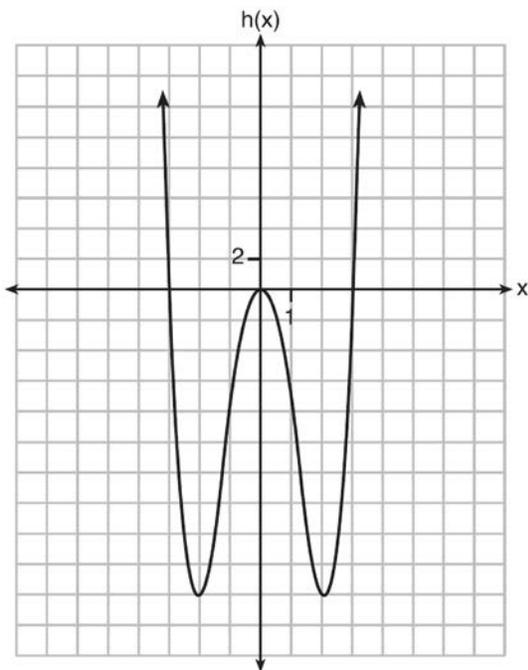
3

15 What is the solution set of the equation
$\frac{3x+25}{x+7} - 5 = \frac{3}{x}$?

1) $\left\{ \frac{3}{2}, 7 \right\}$

2) $\left\{ \frac{7}{2}, -3 \right\}$

3) $\left\{ -\frac{3}{2}, 7 \right\}$

4) $\left\{ -\frac{7}{2}, -3 \right\}$

16 Functions f, g, and h are given below.

$$f(x) = \sin(2x)$$

$$g(x) = f(x) + 1$$

h(x)

Which statement is true about functions f, g, and h?
1) $f(x)$ and $g(x)$ are odd, $h(x)$ is even.
2) $f(x)$ and $g(x)$ are even, $h(x)$ is odd.
3) $f(x)$ is odd, $g(x)$ is neither, $h(x)$ is even.
4) $f(x)$ is even, $g(x)$ is neither, $h(x)$ is odd.

17 The expression $\frac{6x^3 + 17x^2 + 10x + 2}{2x+3}$ equals

1) $3x^2 + 4x - 1 + \frac{5}{2x+3}$

2) $6x^2 + 8x - 2 + \frac{5}{2x+3}$

3) $6x^2 - x + 13 - \frac{37}{2x+3}$

4) $3x^2 + 13x + \frac{49}{2} + \frac{151}{2x+3}$

18 The solutions to the equation $-\frac{1}{2}x^2 = -6x + 20$ are

1) $-6 \pm 2i$
2) $-6 \pm 2\sqrt{19}$
3) $6 \pm 2i$
4) $6 \pm 2\sqrt{19}$

19 What is the completely factored form of
$k^4 - 4k^2 + 8k^3 - 32k + 12k^2 - 48$?
1) $(k-2)(k-2)(k+3)(k+4)$
2) $(k-2)(k-2)(k+6)(k+2)$
3) $(k+2)(k-2)(k+3)(k+4)$
4) $(k+2)(k-2)(k+6)(k+2)$

20 Which statement is *incorrect* for the graph of the
function $y = -3\cos\left[\frac{\pi}{3}(x-4) \right] + 7$?

1) The period is 6.
2) The amplitude is 3.
3) The range is [4,10].
4) The midline is $y = -4$.

21 Algebraically determine the values of x that satisfy
the system of equations below.
$$y = -2x + 1$$
$$y = -2x^2 + 3x + 1$$

22 The results of a poll of 200 students are shown in the table below:

	Preferred Music Style		
	Techno	Rap	Country
Female	54	25	27
Male	36	40	18

For this group of students, do these data suggest that gender and preferred music styles are independent of each other? Justify your answer.

23 For the function $f(x) = (x-3)^3 + 1$, find $f^{-1}(x)$.

24 Given: $h(x) = \dfrac{2}{9}x^3 + \dfrac{8}{9}x^2 - \dfrac{16}{13}x + 2$

$k(x) = -|0.7x| + 5$

State the solutions to the equation $h(x) = k(x)$, rounded to the *nearest hundredth*.

25 Algebraically prove that the difference of the squares of any two consecutive integers is an odd integer.

26 Rewrite the expression
$\left(4x^2 + 5x\right)^2 - 5\left(4x^2 + 5x\right) - 6$ as a product of four linear factors.

27 After sitting out of the refrigerator for a while, a turkey at room temperature (68°F) is placed into an oven at 8 a.m., when the oven temperature is 325°F. Newton's Law of Heating explains that the temperature of the turkey will increase proportionally to the difference between the temperature of the turkey and the temperature of the oven, as given by the formula below:

$$T = T_a + \left(T_0 - T_a\right)e^{-kt}$$

T_a = the temperature surrounding the object

T_0 = the initial temperature of the object

t = the time in hours

T = the temperature of the object after t hours

k = decay constant

The turkey reaches the temperature of approximately 100° F after 2 hours. Find the value of k, to the *nearest thousandth*, and write an equation to determine the temperature of the turkey after t hours. Determine the Fahrenheit temperature of the turkey, to the *nearest degree*, at 3 p.m.

28 Seventy-two students are randomly divided into two equally-sized study groups. Each member of the first group (group 1) is to meet with a tutor after school twice each week for one hour. The second group (group 2), is given an online subscription to a tutorial account that they can access for a maximum of two hours each week. Students in both groups are given the same tests during the year. A summary of the two groups' final grades is shown below:

	Group 1	Group 2
$\bar{x}$	80.16	83.8
S_x	6.9	5.2

Calculate the mean difference in the final grades (group 1 – group 2) and explain its meaning in the context of the problem. A simulation was conducted in which the students' final grades were rerandomized 500 times. The results are shown below.

Simulated Group Mean Differences
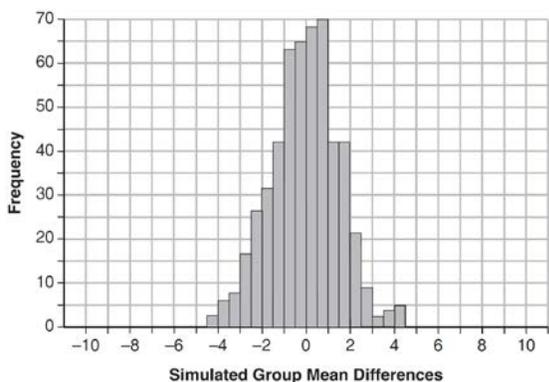
Use the simulation to determine if there is a significant difference in the final grades. Explain your answer.

29 Given $z(x) = 6x^3 + bx^2 - 52x + 15$, $z(2) = 35$, and $z(-5) = 0$, algebraically determine all the zeros of $z(x)$.

30 Two versions of a standardized test are given, an April version and a May version. The statistics for the April version show a mean score of 480 and a standard deviation of 24. The statistics for the May version show a mean score of 510 and a standard deviation of 20. Assume the scores are normally distributed. Joanne took the April version and scored in the interval 510-540. What is the probability, to the *nearest ten thousandth*, that a test paper selected at random from the April version scored in the same interval? Maria took the May version. In what interval must Maria score to claim she scored as well as Joanne?

31 Titanium-44 is a radioactive isotope such that every 63 years, its mass decreases by half. For a sample of titanium-44 with an initial mass of 100 grams, write a function that will give the mass of the sample remaining after any amount of time. Define all variables. Scientists sometimes use the average yearly decrease in mass for estimation purposes. Use the average yearly decrease in mass of the sample between year 0 and year 10 to predict the amount of the sample remaining after 40 years. Round your answer to the *nearest tenth*. Is the actual mass of the sample or the estimated mass greater after 40 years? Justify your answer.