

MCS21 – Calculus  
Exam 4 Review Sheet

Topics:

- the product rule
- the quotient rule
- the chain rule
- higher order derivatives

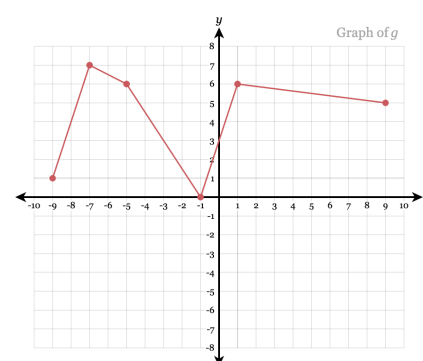
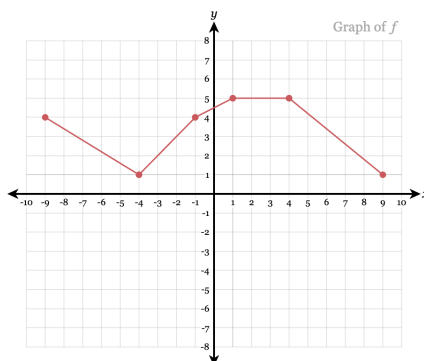
Practice:

1. (a) If  $f(x) = 98x^{99}$ , find  $f^{(100)}(x)$ .  
 (b) If  $g(x) = 50x^{49}$ , find  $g^{(50)}(x)$  and  $g^{(49)}(x)$   
 [Recall that  $n! = n(n-1)(n-2) \cdots 1$ . For example,  $5! = 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$ .]
2. Given  $h(x) = 9x^{30} - 19x^{100} + 14x - 17$ .  
 (a) If  $h^{(n)}(x)$  is a single term, what is the smallest value of  $n$ ?  
 (b) If  $h^{(k)}(x)$  is a constant, what is the smallest value of  $k$ ?  
 (c) If  $h^{(m)}(x)$  is equal to zero, what is the smallest value of  $m$ ?
3. If  $f(x) = \sqrt{(x^2 + 2)^5}$ , then find  $f'(x)$ .
4. If  $y = \frac{5}{x^7}$ , find  $y'''$ .
5. If  $f(x) = (x-2)^3(5-4x^2)^2$ , then find  $f'(0)$ .
6. If  $f(x) = x^2 \cdot \sqrt[3]{3x+1}$ , then find  $f'(x)$ .
7. If  $y = \frac{(x+5)^{10}}{\sqrt{x-3}}$ , then find  $\frac{dy}{dx}$ .
8. If  $y = \frac{2-x}{5x-1}$ , then find  $\frac{d^2y}{dx^2}$ .
9. Find  $\left. \frac{dy}{dx} \right|_{x=3}$  if  $y = \sqrt[3]{x^2 - 1}$ .
10. If  $y = (x-5)^3(8-3x)^4$ , then find  $\frac{dy}{dx}$  in completely factored form.
11. Suppose that  $f(6) = 3$  and  $f'(6) = 4$ .

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| (a) Find $h'(2)$ if $h(x) = f(3x)$ .<br>(b) Find $h'(6)$ if $h(x) = [f(x)]^3$ .<br>(c) Find $h'(6)$ if $h(x) = \sqrt{f(x)}$ . | (d) Find $h'(36)$ if $h(x) = \frac{2f(\sqrt{x})}{x}$ .<br>(e) Find $h'(3)$ if $h(x) = f(2x) \cdot (3x^3 - 4)$ . |
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12. Use the graphs at the right to find:

- (a)  $h'(-3)$  if  $h(x) = f(g(x))$ .
- (b)  $h'(5)$  if  $h(x) = g(f(x))$ .
- (c)  $h'(0)$  if  $h(x) = f(2x)$ .
- (d)  $h'(0)$  if  $h(x) = g(f(3x))$ .



13. The functions  $f$  and  $g$  have continuous second derivatives. The accompanying table gives the values of the functions and their derivatives at several values of  $x$ .

- (a) If  $h(x) = \frac{g(x)}{f(x)}$ , find  $h'(1)$ .
- (b) If  $k(x) = f(g(x))$ , find  $k'(3)$ .

$x$	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	-6	3	2	8
2	2	-2	-3	0
3	8	7	6	2
6	4	5	3	-1