The Second Derivative Test

Do Now:

Find all values of x at which the function $f(x) = x^4 - 4x^2$ has a relative extremum. Determine whether each value of x is a relative maximum or relative minimum.

The First Derivative Test for Relative Extrema

Theorem Let c be a critical number of a function f and (a, b) is an open interval containing c. If f is continuous on the closed interval $[a, b]$ and differentiable on the open interval (a, b), except possibly at c, then:

1. If $f'(x) > 0$ for all x in the interval (a, c) and $f'(x) < 0$ for all x in the interval (c, b), then $f(c)$ is a relative maximum of the function f.
2. If $f'(x) < 0$ for all x in the interval (a, c) and $f'(x) > 0$ for all x in the interval (c, b), then $f(c)$ is a relative minimum of the function f.

The Second Derivative Test for Relative Extrema

Theorem Let c be a critical number of a function f. If f is differentiable on an open interval containing c, then:

1. If $f''(c) < 0$, then f has a relative maximum occurs at $x = c$ and the relative maximum is $f(c)$,
2. If $f''(c) > 0$, then f has a relative minimum occurs at $x = c$ and the relative minimum is $f(c)$,
3. If $f''(c) = 0$, then the test fails and you will need to use the First Derivative Test to classify whether a relative maximum, relative minimum, or neither occurs at $x = c$.

NOTE: In general, use the Second Derivative Test if the second derivative of the function can be calculated *quickly* and you do not need to find where the function is increasing or decreasing.

Find the critical numbers for the each function and determine whether a relative maximum, relative minimum, or neither occurs at each value. [Use the Second Derivative Test whenever it is convenient.]

1. $f(x) = x^2$

2. $f(x) = -x^2$

3. $f(x) = x^2 - 5x + 4$

4. $f(x) = -2x^2 + 12x - 19$

5. $f(x) = \dfrac{6}{4x^2 + 3x - 22}$

6. $f(x) = 2x^3 - 13x^2 - 20x + 18$

7. $f(x) = x^4 + 8x^3 - 12$

8. $f(x) = 5x - \dfrac{4}{x^2}$

9. $f(x) = \dfrac{4}{x^2 - 1}$

10. $f(x) = \dfrac{5x - 17}{9 - x^2}$

11. $f(x) = (x + 5)^{\frac{1}{4}}$

12. $f(x) = 3x^5 - 20x^3$

13. $f(x) = (x - 2)^5$

14. $f(x) = x^3 - 3x + 2$

15. $f(x) = 3(x + 7)^{\frac{4}{3}}$