Problems Involving Continuity

In 1 – 4, sketch the graph of a function f that satisfies the stated conditions.

1. f has a limit at $x = 3$, but it is not continuous at $x = 3$.

2. f is not continuous at $x = 3$, but if its value at $x = 3$ is changed from $f(3) = 1$ to $f(3) = 0$, f becomes continuous at $x = 3$.

3. f has a removable discontinuity at $x = c$ for which $f(c)$ is undefined.

4. f has a removable discontinuity at $x = c$ for which $f(c)$ is defined.

In 5 – 6, use the definition of continuity to prove that the function is discontinuous at the given value of a.

5. $f(x) = \dfrac{x^2 - 5x + 4}{x - 1}$, $a = 1$

6. $f(x) = \begin{cases} \dfrac{x^2 - 5x + 4}{x - 1} & \text{if } x \neq 3 \\ 1 & \text{if } x = 3 \end{cases}$, $a = 3$

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In 7 – 9, use the definition of continuity to find the values of k and m, if possible, that will make the function continuous everywhere.

7. $$f(x)=\begin{cases} mx-1, & x<-1 \\ -x^2+1, & -1\le x<2 \\ \frac{1}{2}x+k, & x\ge 2 \end{cases}$$

8. $$f(x)=\begin{cases} x^2+5, & x>2 \\ m(x+3)+k, & -1<x\le 2 \\ 2x^3+x+7, & x\le -1 \end{cases}$$

9. $$f(x)=\begin{cases} x-2, & x\le -1 \\ kx-mx^2, & -1<x<1 \\ -2x+3, & x\ge 1 \end{cases}$$