

Aim: How do we “reduce” trig function values to trig functions of positive acute angles?

I. Do Now:

1. Find the exact value of each:

(a) $\sin \frac{\pi}{3}$

(b) $\sin \frac{2\pi}{3}$

(c) $\sin \frac{4\pi}{3}$

(d) $\sin \frac{5\pi}{3}$

(e) $\cos 45^\circ$

(f) $\cos 135^\circ$

(g) $\cos 225^\circ$

(h) $\cos 315^\circ$

2. Find the exact values of all six trigonometric functions of θ if the point $(-8, -11)$ is on the terminal side of θ .

II. “Reducing” Trig Functions of Any Angle to a Trig Function of a Positive Acute Angle

Due to the circular nature of trig functions (they are sometimes referred to as circular functions), the value of a trig function at a given angle is always the same as its value at that angle’s reference angle, except when there is a variation in sign. Using A-S-T-C, we can “reduce” any trig function value to a trig function of its reference angle. (Note: Using cofunctions, any trig function can be further “reduced” to a function of a positive acute angle less than 45° .)

Examples:

$$\sin 240^\circ = \ominus \sin 60^\circ$$

$$= -\sin 60^\circ$$

$$= -\frac{\sqrt{3}}{2}$$

$$\cos 300^\circ = \oplus \cos 60^\circ$$

$$= \cos 60^\circ$$

$$= \frac{1}{2}$$

III. Express as a function of a positive acute angle and find the exact value without using a calculator:

1. $\sin 120^\circ$

2. $\cos 210^\circ$

3. $\tan 135^\circ$

4. $\sec \frac{4\pi}{3}$

5. $\tan \frac{5\pi}{4}$

6. $\csc \frac{5\pi}{6}$

7. $\cot \frac{2\pi}{3}$

8. $\cos \frac{5\pi}{3}$

9. $\sin \frac{17\pi}{3}$

10. $\csc \frac{7\pi}{6}$

11. $\tan\left(-\frac{13\pi}{6}\right)$

12. $\cot\left(-\frac{15\pi}{4}\right)$

IV. Practice with Quadrantal Angles: Evaluate:

13. $\sin 90^\circ$

14. $\cos 180^\circ$

15. $\tan 360^\circ$

16. $\csc 270^\circ$

17. $\cot \frac{\pi}{2}$

18. $\sec 7\pi$

19. $\sin\left(-\frac{11\pi}{2}\right)$

20. $\tan 3\pi$

V. More Practice:

Express the given function as a function of a positive acute angle and, if possible, find the exact value:

21. $\csc 1,200^\circ$

22. $\tan 345^\circ$

23. $\cos 208^\circ$

24. $\sin(-250^\circ)$

25. $\sec \frac{50\pi}{3}$

26. $\csc \frac{13\pi}{7}$

27. $\sin\left(-\frac{147\pi}{4}\right)$

28. $\cos \frac{19\pi}{5}$

Find the exact value of the given expression:

29. $\left(\sec \frac{2\pi}{3}\right)\left(\sin \frac{2\pi}{3}\right)$

30. $\csc \frac{\pi}{4} + \cot \frac{3\pi}{4} + \sec 5\pi$

31. $\tan \frac{\pi}{3} + \cot \frac{5\pi}{6}$