

Aim: How do we find the n th roots of a complex number? (Day 2)**I. Do Now:**

- Find all four fourth roots of 1 and represent them graphically on the complex plane.

The n th Roots of Unity

A complex number $v = a + bi$ is an n th root of z if $v^n = z$. If $z = 1$, then v is called an n th root of unity. Since $z = 1 = 1 + 0i$ is equivalent to $\cos 0 + i \sin 0$ in trigonometric form, the n th roots of unity are $\cos \frac{2\pi k}{n} + i \sin \frac{2\pi k}{n}$ for $k = 0, 1, 2, \dots, n-1$.

II. n th Roots of a Complex Number

For a positive integer n , the complex number $z = r(\cos \theta + i \sin \theta)$ has exactly n distinct n th roots given by $\sqrt[n]{r} \left[\cos \left(\frac{\theta + 2\pi k}{n} \right) + i \sin \left(\frac{\theta + 2\pi k}{n} \right) \right]$, where $k = 0, 1, 2, \dots, n-1$.

There are always n n th roots of a number, if we include all complex roots. If all roots of a number are graphed on the complex plane, they will be located $\frac{360^\circ}{n}$ (or $\frac{2\pi}{n}$ radians) apart.

To find all of them quickly:

- Convert the number z that you wish to find roots of to trigonometric ($r \operatorname{cis} \theta$) form.
- Find the 1st n th root of z by taking the n th root of the modulus and by multiplying the argument by $\frac{1}{n}$.
- Find all other roots by adding multiples of $\frac{2\pi}{n}$ (or $\frac{360^\circ}{n}$) to the argument of the 1st root.

III. Applications

- Solve the equation $x^6 = 1$.
(i.e., find the sixth roots of unity)

- Find the fourth roots of $3 + 3i\sqrt{3}$.
Leave your answers in trigonometric form.

- Find the sixth roots of 64
in $a + bi$ form.

- Find the fifth roots of unity.
Leave your answers in trigonometric form.

- Solve the equation $x^5 + 243 = 0$
for all complex roots.
Leave your answers in trigonometric form.

- Solve the equation $x^3 = -4\sqrt{2} - 4i\sqrt{2}$
for all complex roots.
Leave your answers in trigonometric form.

HW41

- Read pages 449 – 451.
- p. 452: 70, 75, 119, 127 (hint: raise it to the 6th power),
141 (leave in trig form), 152 (leave in trig form), 158, 159