Name: ________________________ **Precalculus Unit 6, Lesson 8**

Aim: How do we find the inverse of a square matrix?

I. Do Now:

1. Given $A=\begin{bmatrix} -1 & 2 \\ -1 & 1 \end{bmatrix}$ and $B=\begin{bmatrix} 1 & -2 \\ 1 & -1 \end{bmatrix}$. Find AB and BA.

II. Definition:

If A is a square matrix, and if there exists A^{-1} (the inverse of A), such that $A\cdot A^{-1}=A^{-1}\cdot A=\text{Identity}$, then A^{-1} is the inverse of A.

- If a matrix has an inverse, it is called an *invertible matrix*.
- If a matrix does not have an inverse, it is called a *singular matrix.*
- To verify that A and B are inverses of each other, multiply $A\cdot B$ and $B\cdot A$ and confirm that you obtain the identity matrix.

III. There are several methods to find the inverse of a matrix.

Example: Find the inverse of $A=\begin{bmatrix} 1 & 4 \\ -1 & -3 \end{bmatrix}$.

i. Solve the equation $AX=I$ for X, where I is the identity matrix:

$AX=I$

$$\begin{bmatrix} 1 & 4 \\ -1 & -3 \end{bmatrix}\begin{bmatrix} x_{11} & x_{12} \\ x_{21} & x_{22} \end{bmatrix}=\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \Longrightarrow \begin{bmatrix} 1x_{11}+4x_{21} & 1x_{12}+4x_{22} \\ -1x_{11}-3x_{21} & -1x_{12}-3x_{22} \end{bmatrix}=\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{aligned} 1x_{11}+4x_{21}&=1 \\ 1x_{12}+4x_{22}&=0 \\ -1x_{11}-3x_{21}&=0 \\ -1x_{12}-3x_{22}&=1 \end{aligned} \Longrightarrow$$

Then, solve the system of equations.

ii. Adjoin the original matrix to the identity matrix and use Gauss-Jordan:

$$A=\left[\begin{array}{cc|cc} 1 & 4 & 1 & 0 \\ -1 & -3 & 0 & 1 \end{array}\right]$$

$$\left[\begin{array}{cc|cc} 1 & 4 & 1 & 0 \\ -1 & -3 & 0 & 1 \end{array}\right] \xrightarrow[R_1+R_2]{} \left[\begin{array}{cc|cc} 1 & 4 & 1 & 0 \\ 0 & 1 & 1 & 1 \end{array}\right] \xrightarrow{R_1-4R_2} \left[\begin{array}{cc|cc} 1 & 0 & -3 & -4 \\ 0 & 1 & 1 & 1 \end{array}\right] \Longrightarrow A^{-1}=\begin{bmatrix} -3 & -4 \\ 1 & 1 \end{bmatrix}$$

A^{-1}

iii. For 2×2 matrices only, if $A=\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ and $ad-bc\neq 0$, then the inverse is:

$$A^{-1}=\frac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix} \qquad A^{-1}=\frac{1}{(1)(-3)-(4)(-1)}\begin{bmatrix} -3 & -4 \\ 1 & 1 \end{bmatrix}=\frac{1}{1}\begin{bmatrix} -3 & -4 \\ 1 & 1 \end{bmatrix}=\begin{bmatrix} -3 & -4 \\ 1 & 1 \end{bmatrix}$$

IV. Practice: Use methods (ii) and (iii) (where possible) to find B^{-1} for each of the following:

2. $B=\begin{bmatrix} 3 & -1 \\ -2 & 2 \end{bmatrix}$

3. $B=\begin{bmatrix} -3 & 2 \\ 7 & 4 \end{bmatrix}$

4. $B=\begin{bmatrix} 2 & 1 \\ 6 & 3 \end{bmatrix}$

5. $B=\begin{bmatrix} 2 & 3 & 1 \\ 3 & 3 & 1 \\ 2 & 4 & 1 \end{bmatrix}$

HW51
- Read pages 532 – 536.
- p. 538: 5, 6, 11, 12, 29
- p. 528: 36, 44
- p. 515: 70