Aim: How do we multiply matrices?

I. Do Now: Multiplying a matrix by a real number (scalar) is easy; we just multiply each entry by the scalar. Multiplying a matrix by another matrix may seem unusual at first, but the definition has many practical applications. Refer to the example below and try to fill in the remaining three entries.

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \times \begin{bmatrix} 7 & 8 \\ 9 & 10 \\ 11 & 12 \end{bmatrix} = \begin{bmatrix} 58 \\ \end{bmatrix}$$

II. Development:

Given
$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$
, $B = \begin{bmatrix} 0 & 1 \\ 2 & 3 \end{bmatrix}$, and $C = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, find each product:

(a) AB (b) BA (c) BC (d) CB

What conclusions can you draw from the answers above?

Important Note:

To multiply an **m**×**n** matrix by an $\mathbf{n} \times \mathbf{p}$ matrix, the \mathbf{n} 's must be the same, and the result is an $\mathbf{m} \times \mathbf{p}$ matrix.

 $m \times n \times n \times p \rightarrow m \times p$

III. **Additional Practice:**

Multiply, if possible.

$$1. \quad \left[\begin{array}{cc} 1 & 2 \\ 1 & 1 \end{array} \right] \left[\begin{array}{cc} -1 & 2 \\ 1 & -1 \end{array} \right]$$

$$2. \quad \left[\begin{array}{ccc} 1 & -2 & -3 \end{array} \right] \left[\begin{array}{ccc} 2 \\ -1 \\ 1 \end{array} \right]$$

$$1. \begin{bmatrix} 1 & 2 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} -1 & 2 \\ 1 & -1 \end{bmatrix} \qquad 2. \begin{bmatrix} 1 & -2 & -3 \end{bmatrix} \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix} \qquad 3. \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix} \begin{bmatrix} 1 & -2 & -3 \end{bmatrix}$$

$$4. \begin{bmatrix}
-2 & 1 \\
1 & -3 \\
1 & 4
\end{bmatrix}
\begin{bmatrix}
-2 & 3 & 1 & 4 \\
0 & 1 & -1 & 2 \\
2 & -1 & 0 & 1
\end{bmatrix}$$

$$5. \begin{bmatrix}
1 & 3 & 5 \\
6 & 4 & 2 \\
9 & 8 & 7
\end{bmatrix}
\begin{bmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1
\end{bmatrix}$$

$$5. \quad \begin{bmatrix} 1 & 3 & 5 \\ 6 & 4 & 2 \\ 9 & 8 & 7 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

6. Two softball teams submit equipment lists to their sponsors.

	Women's Team	Men's Team
Bats	12	15
Balls	45	38
Gloves	15	17

Each bat costs \$48, each ball costs \$4, and each glove costs \$42. Use matrices to find the total cost of equipment for each team.