

Aim: What is the exact connection between the degree of a polynomial function and the number of its roots

I. Do Now:

1. Divide (by long division):

$$\frac{4x^3 - 7x^2 - 11x + 5}{4x + 5}$$

2. Find all complex roots:

(a) $f(x) = x - 4$ (b) $f(x) = x^2 - 8x + 16$ (c) $f(x) = x^3 + 9x$

II. Important Theorems

The Fundamental Theorem of Algebra (This theorem was proved by Carl Friedrich Gauss (1777 – 1855).) Every polynomial of degree n , where $n > 0$, has at least one root in the complex number system.

The Linear Factorization Theorem

An n th degree polynomial function has exactly n linear factors and therefore, exactly n roots in the complex number system.

Conjugate Pair Theorem

If a complex number $a + bi$ is a zero of a polynomial with real coefficients, then the complex conjugate $a - bi$ is also a zero of the polynomial.

(e.g., if $2 + 3i$ is a zero, then $2 - 3i$ is a zero; if $0 + 2i$ is a zero, then $0 - 2i$ is a zero.)

III. Applications3. Find a fourth degree polynomial function with roots -1 , -1 , and $3i$.5. If $1 + 3i$ is a zero of $f(x) = x^4 - 3x^3 + 6x^2 + 2x - 60$, find all zeros.4. Find a third degree polynomial function with roots 2 and $4 + i$.