MPS21 – Precalculus
Exam 4 Review Sheet

Topics:

- vertex form of a quadratic equation $y = a(x-h)^2 + k$ (completing the square)
- solving optimization (max/min) problems
- graphing higher degree polynomial functions
- end behavior and multiplicities of roots
- The Intermediate Value Theorem
- testing for upper bounds and lower bounds
- Fundamental Theorem of Algebra, Linear Factorization Theorem, and Conjugate Pair Theorem

Practice Problems:

1. (a) Determine the end behavior of the graph of $f(x) = 132x - 12x^2$

   (b) A polynomial function whose range is all real numbers has a relative minimum at (–3, –5) and a relative maximum at (8, 2). Which of the following could be the leading term of this function?
   (i) $-x^3$ (ii) x^3 (iii) x^4 (iv) $-x^4$

2. Write the equation $f(x) = 15 + 96x - 16x^2$ in vertex form by completing the square. State the coordinates of the vertex. Show all steps.

3. Factor the polynomial function $f(x) = x^5 - 13x^3 + 36x$ and find all of its roots.

4. Write the equation of a third degree polynomial with integral coefficients whose roots include -4 and $2 - 3i$.

5. Completely factor $x^4 + 3x^3 + 9x^2 + 27x$ over the complex numbers.

6. If $1 + 3i$ is a zero of $f(x) = x^4 - 3x^3 + 6x^2 + 2x - 60$, find all zeros.

7. Write the equation of a parabola in vertex form with vertex (2, 3) and that passes through the point (–1, 7). Show all steps.

8. (a) List all the *possible* rational roots of $f(x) = 7x^3 - 11x^2 - 3x + 4$

   (b) Which of the possible rational roots of $f(x)$ is an *actual root*? (Use the graph to help you.)

   (c) Use synthetic division with the root you found in (b) to factor the polynomial and then find the other two roots in simplest radical form.

9. (a) Use synthetic division to determine whether –2 is a lower bound for the zeros of $f(x) = x^5 - 3x^4 - 6x^3 + 5x + 5$. Show your work and explain your conclusion.

(b) Use synthetic division to determine whether 5 is an upper bound for the zeros of $f(x) = x^5 - 3x^4 - 6x^3 + 5x + 5$. Show your work and explain your conclusion.

10. An open box is to be made from a 16 inch by 9 inch rectangular sheet of metal by cutting equal squares from the corners and turning up the sides.
(a) Use *x* to represent the sides of the squares. Draw a diagram showing the squares to be removed from the original piece of metal. Write the dimensions of the open box.
(b) Write a function V, the volume of the box, as a function of *x*.
(c) State the domain of this function.
(d) Using your graphing calculator, graph the function and determine the maximum volume of the open box to the nearest hundredth of a cubic inch.
(e) What are the dimensions of the box (to the nearest hundredth of an inch) that yield the maximum volume?

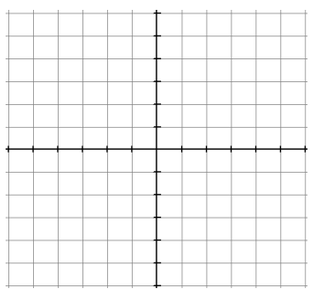
11. Write the equation of a polynomial function of minimum degree with integral coefficients whose roots are $-\frac{2}{3}$, $2-\sqrt{5}$, and $2+\sqrt{5}$.

12. (a) Given a polynomial function $f(x)$, with $f(4) = 5$ and $f(5) = -1$, in what interval does the Intermediate Value Theorem guarantee there will be a root? Explain your answer.

(b) Show that there must be a root of the equation $f(x) = x^4 + x - 3$ in the interval $[1, 2]$.

(c) Given a polynomial function $g(x)$, with $g(-3) = 4$ and $g(-2) = 5$, could there be a root in the interval $[-3, -2]$? Explain your answer.

13. Sketch a reasonable graph of the function $f(x) = x(x+4)^2(x-2)^3$ without using a graphing calculator.



14. Given $f(x) = x^5 + 3x^4 - 11x^3 - 27x^2 + 10x + 24$.
(a) List all of the possible rational zeros.
(b) Find all the roots algebraically.