

MCS21 Homework 26

The Chain Rule

If $y = f(u)$ is a differentiable function of u , and $u = g(x)$ is a differentiable function of x , then

$y = f(g(x))$ is a differentiable function of x and

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} \quad \text{or} \quad \frac{d}{dx}[f(g(x))] = f'(g(x)) \cdot g'(x)$$

1. Suppose that $w = u(v(x))$ and $u(0) = 1$, $v(0) = 2$, $u'(0) = 3$, $u'(2) = 4$, $v'(0) = 5$, and $v'(2) = 6$. Find $w'(0)$.

2. A table of values for f , g , f' , and g' is given.

- a) If $h(x) = f(g(x))$, find $h'(1)$.

x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
1	3	2	4	6
2	1	8	5	7
3	7	2	7	9

- b) If $H(x) = g(f(x))$, find $H'(1)$.

- c) If $F(x) = f(f(x))$, find $F'(2)$.

- d) If $G(x) = g(g(x))$, find $G'(3)$.

3. Find $\frac{d^2y}{dx^2}$.

(a) $y = (x^3 - 8)^6$

(b) $y = 3(x^2 - 18)^{50}$

4. Find $f'''(x)$ if $f(x) = \frac{9}{x^6}$

5. Find the derivative of each function.

(a) $y = \left(\frac{3x-1}{x^2+x}\right)^3$

(b) $y = x^2(4x^3 - 5x)^2$

(c) $y = \frac{(x-3)^4}{x^2+7x-1}$

(d) $y = [(x+4)(2x^3 - 5x)]^6$

(e) $y = x\sqrt{2x+3}$

(f) $y = \frac{3x^2}{\sqrt{x^2+2x-1}}$