

MCS21 – Calculus
Exam 4 Review Sheet

Topics:

- non-differentiability of functions (corner/cusp, points of discontinuity, vertical tangent)
- differentiability of piecewise-defined functions
- the chain rule (incorporates use of power, product, and quotient rules)
- higher order derivatives

Practice:

1. If $f'(a)$ does not exist, which of the following *must* be true?
(A) $f(x)$ is discontinuous at $x = a$. (D) f has a “hole” at $x = a$.
(B) $\lim_{x \rightarrow a} f(x)$ does not exist. (E) None of the above must be true.
(C) f has a vertical tangent at $x = a$.

2. Which statement is true for the function $f(x)$, if $f(x) = \begin{cases} x+1, & \text{if } x \leq 1 \\ x^2+1, & \text{if } x > 1 \end{cases}$?
(A) $f(x)$ is continuous and differentiable at $x = 1$.
(B) $f(x)$ is continuous but non-differentiable at $x = 1$.
(C) $f(x)$ is not continuous but is differentiable at $x = 1$.
(D) $f(x)$ is not continuous and also non-differentiable at $x = 1$.
(E) $f(x)$ has a removable discontinuity and is differentiable at $x = 1$.

3. Let $f(x) = \begin{cases} x^3 + 16 & x < \frac{1}{2} \\ \frac{3}{4}x^2 & x \geq \frac{1}{2} \end{cases}$. Determine whether f is differentiable at $x = \frac{1}{2}$. If so, find the value of the derivative there.

4. Let $f(x) = \begin{cases} x^2 & x \leq 2 \\ mx + b & x > 2 \end{cases}$. Find values of m and b that make f differentiable everywhere.

5. If $f(x) = \sqrt{(x^2 + 2)^5}$, then find $f'(x)$.

6. If $y = \frac{5}{x^7}$, find y''' .

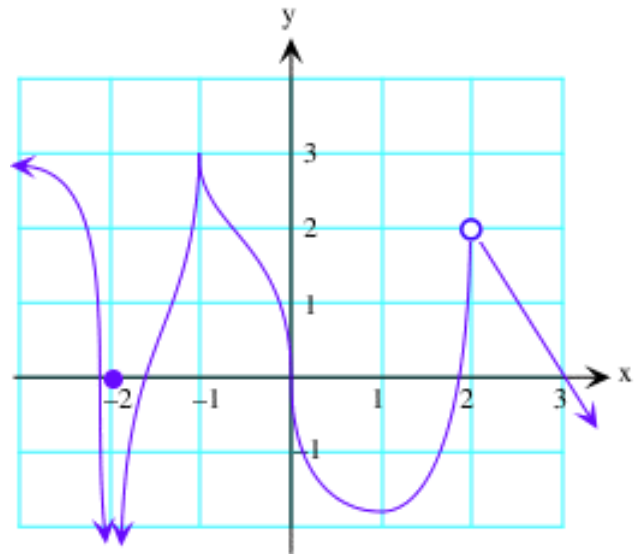
7. If $f(x) = (x - 2)^3(5 - 4x^2)^2$, then find $f'(0)$.

8. If $f(x) = x^2 \cdot \sqrt[3]{3x + 1}$, then find $f'(x)$.

9. If $y = \frac{(x + 5)^{10}}{\sqrt{x - 3}}$, then find $\frac{dy}{dx}$.

10. If $y = \frac{2-x}{5x-1}$, then find $\frac{d^2y}{dx^2}$.

11. Use accompanying graph of $f(x)$ to answer parts (a) through (h) below.



(a) $\lim_{x \rightarrow -2} f(x) = \underline{\hspace{2cm}}$

(b) $f'(3) = \underline{\hspace{2cm}}$

(c) $f(2) = \underline{\hspace{2cm}}$

(d) $f(-2) = \underline{\hspace{2cm}}$

(e) Create a true statement by filling in the box with one of the three symbols: $>$, $<$, or $=$.

$$f'\left(-\frac{1}{2}\right) \square f'\left(\frac{3}{2}\right)$$

(f) State all values of x where $f(x)$ is discontinuous.

(g) State all values of x where $f(x)$ is not differentiable.

(h) The value of $f'(x)$ is zero when:

(A) $x = -1$

(B) $x = 0$

(C) $x = 1$

(D) $x > 2$

(E) $x < -1$

12. Find $\left.\frac{dy}{dx}\right|_{x=3}$ if $y = \sqrt[3]{x^2 - 1}$.

13. If $y = (x - 5)^3(8 - 3x)^4$, then find $\frac{dy}{dx}$ in completely factored form.

14. Let $f(x) = \begin{cases} 3bx^3 - 2 & x \geq 1 \\ x^2 - ax^4 & x < 1 \end{cases}$. Find the values of a and b such that $f(x)$ is differentiable at $x = 1$.

15. Suppose that $f(6) = 3$ and $f'(6) = 4$.

(a) Find $h'(2)$ if $h(x) = f(3x)$.

(d) Find $h'(36)$ if $h(x) = \frac{2f(\sqrt{x})}{x}$.

(b) Find $h'(6)$ if $h(x) = [f(x)]^3$.

(e) Find $h'(3)$ if $h(x) = f(2x) \cdot (3x^3 - 4)$.

(c) Find $h'(6)$ if $h(x) = \sqrt{f(x)}$.