Name: ___________________________________ Date: _________________

SOLVING QUADRATIC EQUATIONS WITH COMPLEX SOLUTIONS
COMMON CORE ALGEBRA II

As we saw in the last unit, the roots or zeroes of any quadratic equation can be found using the quadratic formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Since this formula contains a square root, it is fair to investigate solutions to quadratic equations now when the quantity $b^2 - 4ac$, known as the **discriminant**, is negative. Up to this point, we would have concluded that if the discriminant was negative, the quadratic had no (real) solutions. But, now it can have **complex solutions**.

***Exercise* #1:** Use the quadratic formula to find all solutions to the following equation. Express your answers in simplest $a + bi$ form.

$$x^2 - 4x + 29 = 0$$

As long as our solutions can include complex numbers, then any quadratic equation can be solved for two roots.

***Exercise* #2:** Solve each of the following quadratic equations. Express your answers in simplest $a + bi$ form.

(a) $x^2 - 5x + 30 = 7x - 10$

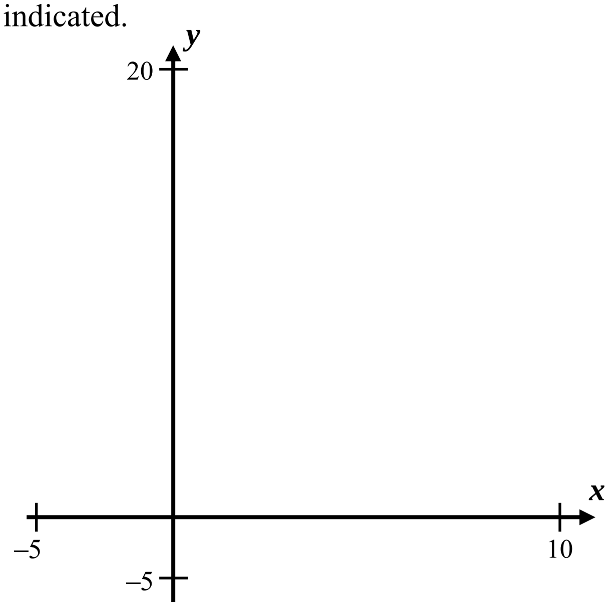
(b) $x^2 + 16x + 15 = 10x + 4$

There is an interesting connection between the x-intercepts (zeroes) of a parabola and complex roots with non-zero imaginary parts. The next exercise illustrates this important concept.

***Exercise* #3:** Consider the parabola whose equation is $y = x^2 - 6x + 13$.

(a) Algebraically find the x-intercepts of this parabola. Express your answers in simplest $a + bi$ form.

(b) Using your calculator, sketch a graph of the parabola on the axes below. Use the window indicated.



(c) From your answers to (a) and (b), what can be said about parabolas whose zeroes are complex roots with non-zero imaginary parts?

***Exercise* #4:** Use the **discriminant** of each of the following quadratics to determine whether it has x-intercepts.

(a) $y = x^2 - 3x - 10$

(b) $y = x^2 + 6x + 10$

(c) $y = 2x^2 + 3x + 5$

***Exercise* #5:** Which of the following quadratic functions, when graphed, would not cross the x-axis?

(1) $y = 2x^2 + 5x - 3$

(3) $y = 4x^2 - 4x + 5$

(2) $y = -x^2 - x + 6$

(4) $y = 3x^2 - 13x + 4$

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SOLVING QUADRATIC EQUATIONS WITH COMPLEX SOLUTIONS
COMMON CORE ALGEBRA II HOMEWORK

FLUENCY

1. Solve each of the following quadratic equations. Express your solutions in simplest $a+bi$ form. Check.

(a) $x^2+4x+20=12x-5$

(b) $x^2=x-1$

(c) $2x^2-25x+27=-15x-10$

(d) $8x^2+36x+24=12x+5$

(e) $x^2+6x+15=8x-2$

(f) $4x^2+38x+50=10x-35$

2. Which of the following represents the solution set to the equation $x^2 - 2x + 2 = 0$?

(1) $x = -1$ or 2 (3) $x = 2 \pm i$

(2) $x = 1 \pm 2i$ (4) $x = 1 \pm i$

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3. The solutions to the equation $x^2 + 6x + 11 = 0$ are

(1) $x = -3 \pm i\sqrt{2}$ (3) $x = -6 \pm i\sqrt{11}$

(2) $x = -3 \pm 2i\sqrt{2}$ (4) $x = -6 \pm 2i\sqrt{11}$

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4. Using the discriminant, $b^2 - 4ac$, determine whether each of the following quadratics has real or imaginary zeroes.

(a) $y = 2x^2 - 7x + 6$ (b) $y = 3x^2 + 2x + 1$ (c) $y = x^2 - 8x + 14$

(d) $y = 2x^2 - 12x + 26$ (e) $y = -2x^2 + 6x - 5$ (f) $y = 4x^2 - 4x + 1$

5. Which of the following quadratics, if graphed, would lie entirely above the *x*-axis? Try to use the discriminant to solve this problem and then graph to check.

(1) $y = 2x^2 + x - 21$ (3) $y = x^2 - 4x + 7$

(2) $y = x^2 - x - 6$ (4) $y = x^2 - 10x + 16$

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REASONING

6. For what values of *c* will the quadratic $y = x^2 + 6x + c$ have no real zeroes? Set up and solve an inequality for this problem.