Name: ___________________________________ Date: _________________

COMPLEX NUMBERS
COMMON CORE ALGEBRA II

All numbers fall into a very broad category known as complex numbers. Complex numbers can always be thought of as a combination of a real number with an imaginary number and will have the form:

$$a + bi \quad \text{where } a \text{ and } b \text{ are real numbers}$$

We say that a is the real part of the number and bi is the imaginary part of the number. These two parts, the real and imaginary, **cannot be combined**. Like real numbers, complex numbers may be added and subtracted. The key to these operations is that real components can combine with real components and imaginary with imaginary.

***Exercise* #1:** Find each of the following sums and differences.

(a) $(-2+7i)+(6+2i)$ (b) $(8+4i)+(12-i)$ (c) $(5+3i)-(2-7i)$ (d) $(-3+5i)-(-8+2i)$

***Exercise* #2:** Which of the following represents the sum of $(6+2i)$ and $(-8-5i)$?

(1) $5i$ (3) $2+3i$

(2) $-2-3i$ (4) $-5i$ __________

Adding and subtracting complex numbers is straightforward because the process is similar to combining algebraic expressions that have like terms. The complex numbers are **closed under addition and subtraction**, i.e. when you add or subtract two complex numbers the results is a complex number as well. But, is multiplication closed?

***Exercise* #3** Find the following products. Write each of your answers as a complex number in the form $a+bi$.

(a) $(3+5i)(7+2i)$ (b) $(-2+6i)(3-2i)$ (c) $(4+i)(-5-3i)$

***Exercise* #4:** Consider the more general product $(a+bi)(c+di)$ where constants *a*, *b*, *c* and *d* are real numbers.

(a) Show that the real component of this product will always be $ac-bd$.

(b) Show that the product of $2+3i$ and $4-6i$ results in a purely real number.

(c) Under what conditions will the product of two complex numbers always be a purely imaginary number? Check by generating a pair of complex numbers that have this type of product.

***Exercise* #5:** Determine the result of the calculation below in simplest $a+bi$ form.

$$(5+2i)(-3+i)+4i(2+3i)$$

***Exercise* #6:** Which of the following products would be a purely real number?

(1) $(4+2i)(3-i)$

(3) $(5+2i)(5-2i)$

(2) $(-3+i)(-2+4i)$

(4) $(6+3i)(6+3i)$

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Name: ___________________________________ Date: _________________

COMPLEX NUMBERS
COMMON CORE ALGEBRA II HOMEWORK

FLUENCY

1. Find each of the following sum or difference.

(a) $(6+3i)+(-2+9i)$ (b) $(-7+i)-(3+5i)$ (c) $(10-3i)+(6-8i)$

(d) $(-2+7i)-(15-6i)$ (e) $(15+2i)+(5-5i)$ (f) $(-1+i)-(-5-6i)$

2. Which of the following is equivalent to $3(5+2i)-2(3-6i)$?

(1) $9+18i$ (3) $9-6i$

(2) $21+8i$ (4) $21-2i$

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3. Find each of the following products in simplest $a+bi$ form.

(a) $(5-2i)(-1+7i)$ (b) $(3+9i)(2+4i)$ (c) $(-4-i)(-2+6i)$

4. Complex conjugates are two complex numbers that have the form $a+bi$ and $a-bi$. Find the following products of complex conjugates:

(a) $(5-7i)(5+7i)$ (b) $(10+i)(10-i)$ (c) $(-3+8i)(-3-8i)$

(d) What's true about the product of two complex conjugates?

5. Show that the product of $a+bi$ and $a-bi$ is the purely real number a^2+b^2.

6. The product of $(-8+2i)$ and its conjugate is equal to

(1) $64+4i$ (3) 68

(2) 60 (4) $60-4i$

7. The complex computation $(6+2i)(6-2i)-(3-4i)(3+4i)$ can be simplified to

(1) 15 (3) -10

(2) 39 (4) -35

8. Perform the following complex calculation. Express your answer in simplest $a+bi$ form.

$$(8+5i)(3+2i)-(4+i)(4-i)$$

9. Perform the following complex calculation. Express your answer in simplest $a+bi$ form.

$$7(3-5i)+(4-2i)(-6+7i)$$

10. Simplify the following complex expression. Write your answer in simplest $a+bi$ form.

$$(5+2i)^2+(2-i)^2$$