

Name: \_\_\_\_\_

Date: \_\_\_\_\_

## IMAGINARY NUMBERS COMMON CORE ALGEBRA II



Recall that in the Real Number System, it is not possible to take the square root of a negative quantity because whenever a real number is squared it is non-negative. This fact has a ramification for finding the  $x$ -intercepts of a parabola, as *Exercise #1* will illustrate.

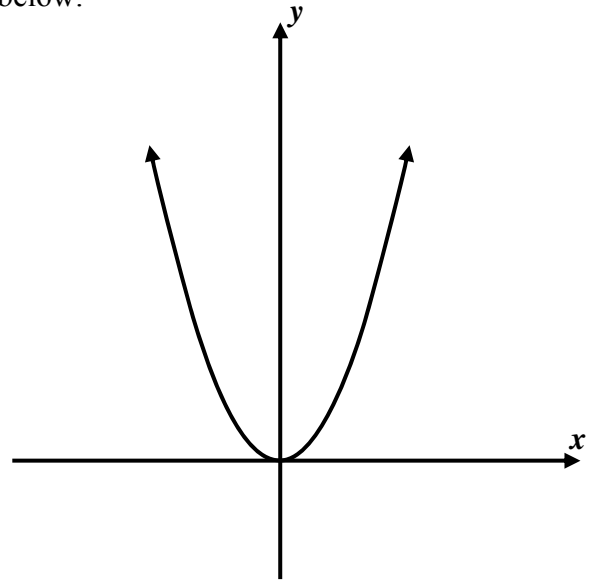
**Exercise #1:** On the axes below, a sketch of  $y = x^2$  is shown. Now, consider the parabola whose equation is given in function notation as  $f(x) = x^2 + 1$ .

(a) How is the graph of  $y = x^2$  shifted to produce the graph of  $f(x)$ ?

(b) Create a quick sketch of  $f(x)$  on the axes below.

(c) What can be said about the  $x$ -intercepts of the function  $y = f(x)$ ?

(d) Algebraically, show that these intercepts do not exist, in the Real Number System, by solving the incomplete quadratic  $x^2 + 1 = 0$ .



Since we cannot solve this equation using Real Numbers, we introduce a new number, called  $i$ , the basis of **imaginary numbers**. Its definition allows us to now have a result when finding the square root of a negative real number. Its definition is given below.

### THE DEFINITION OF THE IMAGINARY NUMBER $i$

$$i = \sqrt{-1}$$

**Exercise #2:** Simplify each of the following square roots in terms of  $i$ .

(a)  $\sqrt{-9}$

(b)  $\sqrt{-100}$

(c)  $\sqrt{-32}$

(d)  $\sqrt{-18}$



**Exercise #2:** Solve each of the following incomplete quadratics. Place your answers in simplest radical form.

(a)  $5x^2 + 8 = -12$

(b)  $\frac{1}{2}x^2 + 20 = 2$

(c)  $2x^2 - 10 = -36$

**Exercise #3:** Which of the following is equivalent to  $5i \cdot 6i$  ?

(1)  $30i$

(3)  $-30$

(2)  $11i$

(4)  $-11$

Powers of  $i$  display a remarkable pattern that allow us to simplify large powers of  $i$  into one of 4 cases. This pattern is discovered in *Exercise #4*.

**Exercise #4:** Simplify each of the following powers of  $i$ .

$i^1 = i$

$i^2 =$

$i^3 =$

$i^4 =$

$i^5 =$

$i^6 =$

$i^7 =$

$i^8 =$

We see, then, from this pattern that every power of  $i$  is either  $-1, 1, i,$  or  $-i$ . And the pattern will repeat.

**Exercise #5:** From the pattern of *Exercise #4*, simplify each of the following powers of  $i$ .

(a)  $i^{38} =$

(b)  $i^{21} =$

(c)  $i^{83} =$

(d)  $i^{40} =$

**Exercise #6:** Which of the following is equivalent to  $5i^{16} + 3i^{23} + i^{26}$  ?

(1)  $8 + 2i$

(3)  $5 - 4i$

(2)  $4 - 3i$

(4)  $2 + 7i$



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**IMAGINARY NUMBERS**  
**COMMON CORE ALGEBRA II HOMEWORK**

**FLUENCY**1. The imaginary number  $i$  is defined as

(1)  $-1$

(3)  $\sqrt{-4}$

(2)  $\sqrt{-1}$

(4)  $(-1)^2$

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2. Which of the following is equivalent to  $\sqrt{-128}$ ?

(1)  $8\sqrt{2}$

(3)  $-8\sqrt{2}$

(2)  $8i$

(4)  $8i\sqrt{2}$

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3. The sum  $\sqrt{-9} + \sqrt{-16}$  is equal to

(1)  $5$

(3)  $7i$

(2)  $5i$

(4)  $7$

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4. Which of the following powers of  $i$  is *not* equal to one?

(1)  $i^{16}$

(3)  $i^{32}$

(2)  $i^{26}$

(4)  $i^{48}$

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5. Which of the following represents all solutions to the equation  $\frac{1}{3}x^2 + 10 = 7$ ?

(1)  $x = \pm 3i$

(3)  $x = \pm i$

(2)  $x = \pm 5i$

(4)  $x = \pm 2i$

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6. Solve each of the following incomplete quadratics. Express your answers in simplest radical form.

(a)  $2x^2 + 100 = -62$

(b)  $\frac{2}{3}x^2 + 20 = 2$



7. Which of the following represents the solution set of  $\frac{1}{2}x^2 - 12 = -37$ ?

(1)  $\pm 7i$                       (3)  $\pm 5i\sqrt{2}$

(2)  $\pm 7i\sqrt{2}$                 (4)  $\pm 3i\sqrt{2}$

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8. Simplify each of the following powers of  $i$  into either  $-1$ ,  $1$ ,  $i$ , or  $-i$ .

(a)  $i^2$                               (b)  $i^3$                               (c)  $i^4$                               (d)  $i^{11}$

(e)  $i^{41}$                               (f)  $i^{30}$                               (g)  $i^{25}$                               (h)  $i^{36}$

(i)  $i^{51}$                               (j)  $i^{45}$                               (k)  $i^{80}$                               (l)  $i^{70}$

9. Which of the following is equivalent to  $i^7 + i^8 + i^9 + i^{10}$ ?

(1)  $1$                                   (3)  $1-i$

(2)  $2+i$                               (4)  $0$

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10. When simplified the sum  $5i^{18} + 7i^{25} + 2i^{28} + 6i^{43}$  is equal to

(1)  $2-4i$                               (3)  $5-7i$

(2)  $-3+i$                               (4)  $8+i$

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11. The product  $(6+2i)(4-3i)$  can be written as

(1)  $24-6i$                               (3)  $2+5i$

(2)  $18+10i$                           (4)  $30-10i$

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