Name: ____________________________________ Date: __________________

The Basic Exponent Properties
Common Core Algebra II

Exponents, which indicate repeated multiplication, are extremely important in higher-level mathematical study because of their importance in numerous areas. The rules they play by, known as the **exponent properties**, are critical to master. We will develop each one of the major seven properties. The order in which these are presented is not unique.

***Exercise* #1 (Property #1):** Consider the expression $x^a \cdot x^b$. How can we rewrite this product equivalently?

(a) Rewrite $x^2 \cdot x^4$. Write as an extended product first if necessary.

(b) Generalize: $x^a \cdot x^b =$

(c) Practice - Rewrite each of the following in simplest form:

(i) $x^{10} \cdot x^3$

(ii) $\left(5x^4\right)\left(6x^3\right)$

(iii) $x^3y^2x^6y$

***Exercise* #2 (Property #2):** Consider the expression $\frac{x^a}{x^b}$. How can we simplify this quotient (division)?

(a) Rewrite the quotient $\frac{x^5}{x^2}$ in simplest form by using the Property #1 and the multiplication property of fractions.

(b) Generalize for $a > b$: $\frac{x^a}{x^b} =$

(c) Practice - Rewrite each of the following in simplest form:

(i) $\frac{x^8}{x^2}$

(ii) $\frac{6x^{10}}{12x^4}$

(iii) $\frac{x^6y^3}{xy^2}$

***Exercise* #3 (Property #3):** But what if the power of the numerator is less than that of the denominator?

(a) Rewrite the quotient $\frac{x^2}{x^5}$ as in *Exercise* #2(a).

(b) What results if you apply Exponent Prop #2?

(c) Generalize: $x^{-a} =$

(d) Rewrite each of the following without the use of negative exponents:

(i) $2^{-3} =$

(ii) $x^{-4} =$

***Exercise* #4 (Property #4):** What if the powers are the same?

(a) Rewrite the quotient $\frac{x^3}{x^3}$ based on the fundamental concept of dividing a quantity by itself.

(b) What results if you apply Exponent Prop #2?

(c) Generalize: $x^0 =$

(d) Simplify each of the following:

(i) $5^0 =$ (ii) $3x^0 =$

***Exercise* #5 (Property #5):** Now let's take a look at the very common scenario of $\left(x^a\right)^b$.

(a) Rewrite $\left(x^2\right)^3$. Write as an extended product first if necessary.

(b) Generalize: $\left(x^a\right)^b =$

(c) Which of the following expressions is *not* equivalent to x^{30}?

(1) $\left(x^{10}\right)^3$

(3) $x^5 \cdot x^6$

(2) $\left(x^6\right)^5$

(4) $x^{10} \cdot x^{20}$

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The final two properties we will look at concern how exponents **distribute** over **multiplication and division**

***Exercise* #6 (Properties #6 and #7):** Let's take a look at $(xy)^a$.

(a) Rewrite $(xy)^3$ using the definition of an exponent along with the associative and commutative properties of multiplication.

(b) Generalize: $(xy)^a =$

(c) Rewrite $\left(\frac{x}{y}\right)^3$ using the definition of an exponent along with the multiplication property of fractions.

(d) Generalize: $\left(\frac{x}{y}\right)^a =$

(e) Rewrite each of the following as equivalent expressions:

(i) $\left(2x^2\right)^3$

(ii) $\left(\frac{3}{x^2}\right)^2$

(iii) $\left(\frac{-2x^2y^5}{3z^3}\right)^3$

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THE BASIC EXPONENT PROPERTIES
COMMON CORE ALGEBRA II HOMEWORK

FLUENCY

1. Express each of the following expressions in "expanded" form, i.e., do all of the multiplication and/or division possible and combine as many exponents as possible.

(a) $x^3 \cdot x^{12}$

(b) $4x^3 \cdot 5x^5$

(c) $\left(-3x^2y\right)\left(5x^7y^3\right)$

(d) $\left(4x^3y^6\right)\left(-7x^4\right)$

(e) $\dfrac{x^9}{x^3}$

(f) $\dfrac{5x^3y^7}{15xy^2}$

(g) $\dfrac{x^3}{x^{10}}$

(h) $\dfrac{10x^4y^3}{25x^8}$

(i) $\left(x^5\right)^8$

(j) $\left(10x^3\right)^0$

(k) $\left(-4x^5\right)^3$

(l) $\left(x^{-2}\right)^4$

2. Which of the following is *not* equal to 2^{-2}? Do *not* use your calculator to do this problem.

(1) $\dfrac{1}{4}$

(3) 0.25

(2) -4

(4) $\dfrac{1}{2^2}$

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3. If the expression $\dfrac{1}{2x}$ was placed in the form ax^b where a and b are real numbers, then which of the following is equal to $a+b$? Show how you arrived at your answer.

(1) 1

(3) $\dfrac{1}{2}$

(2) $\dfrac{3}{2}$

(4) $-\dfrac{1}{2}$

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4. If $f(x)=5x^0+4x^{-3}$ then $f(a)=$

(1) $12a-5$ (3) $\frac{1}{4a^3}+5$

(2) $5+\frac{4}{a^3}$ (4) $-12a+1$

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5. Which of the following is equivalent to $\frac{(4x^8)^3}{(6x^5)^2}$ for all $x\neq 0$? Show the manipulations that lead to your final answer.

(1) $\frac{16}{9}x^{14}$ (3) $\frac{2}{3}x^{14}$

(2) $\frac{16}{9}x^4$ (4) $\frac{2}{3}x^4$

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APPLICATIONS

6. It is helpful to be able to think about very large numbers in terms of powers of 10. You should be familiar with many of these terms, but have you thought about how many 10's are multiplying each other? Here are some numbers to think about and examples of things that would be counted in these quantities. Fill in the proper power of 10. The first has been done for you.

NUMBER	POWER OF 10	EXAMPLE
1 million	$=1,000\cdot 1,000=10^3\cdot 10^3=10^6$	The distance between New York City and Boston is approximately 1 million feet.
1 billion	$=1,000\cdot 1$ million $=10^3\cdot 10^6=$	There are approximately 3 billion seconds in a century.
1 trillion	$=(1\text{ million})^2=(10^6)^2=$	There are 6 trillion miles in a light year, i.e. the distance light can travel in a year.
1 quadrillion	$=1000\cdot 1$ trillion $=$	There are approximately 1 quadrillion ants populating the earth at any time.
1 quintillion	$=(1\text{ billion})^2=$	There are approximately 8 quintillion grains of sand on all of the Earth's beaches.

REASONING

7. The functions $f(x)=2^x$ and $g(x)=8(2)^x$ are both shown graphed. The graph of g is certainly a vertical stretch of the function f by a factor of 8. But, it is also a shift of f by three units left? Can you explain why this is using an exponent law?