

THE NORMAL DISTRIBUTION AND Z-SCORES

COMMON CORE ALGEBRA II



The normal distribution can be used in increments other than half-standard deviations. In fact, we can use either our calculators or tables to determine probabilities (or proportions) for almost any data value within a normally distributed population, as long as we know the population mean, μ , and the population standard deviation, σ . But, first, we will introduce a concept known as a data value's z-score.

THE Z-SCORE OF A DATA VALUE

For a data point x_i , its z-score is calculated by: $z = \frac{x_i - \mu}{\sigma}$. It calculates how far from the mean, in terms of standard deviations, a data point lies. It can be positive if the data point lies above the mean or negative if the data point lies below the mean.

Exercise #1: Boy's heights in seventh grade are normally distributed with a mean height of 62 inches and a standard deviation of 3.2 inches. Find z-scores, rounded to the nearest hundredth, for each of the following heights. Show the calculation that leads to your answer.

(a) $x_i = 66$ inches

(b) $x_i = 57$ inches

(c) $x_i = 70$ inches

Z-scores give us a way to compare how far a data point is away from its mean in terms of standard deviations. We should be able to compute a z-score for a data value and go in the opposite direction.

Exercise #2: Jeremiah took a standardized test where the mean score was a 560 and the standard deviation was 45. If Jeremiah's score resulted in a z-value of 1.84, then what was Jeremiah's score to the nearest whole number?

With z-scores, we can then determine the probability that a subject picked from a normally distributed population would have a characteristic in a certain range. Z-score tables come in many different varieties. The one that comes with this lesson shows only the right hand side, so symmetry will have to be used to determine probabilities.

Exercise #3: The lengths of full grown sockeye salmon are normally distributed with a mean of 29.2 inches and a standard deviation of 2.4 inches.

(a) Find z-scores for sockeye salmon whose lengths are 25 inches to 32 inches. Round to the nearest hundredth.

(b) Use the z-score table to determine the proportion of the sockeye salmon population, to the nearest percent, that lies between 25 inches and 32 inches. Illustrate your work graphically.



Exercise #4: If the scores on a standardized test are normally distributed with a mean of 560 and a standard deviation of 75. Answer the following questions by using z-scores and the normal distribution table.

- (a) Find the probability that a test picked at random would have a score larger than 720. Round to the nearest tenth of a percent.
- (b) Find the probability that a completed test picked at random would have a score less than 500. Round to the nearest tenth of a percent.
- (c) Find the probability that a completed test picked at random would have a score between 500 and 600.
- (d) Find the probability that a completed test picked at random would have a score between 600 and 700.

This process is sometimes used to determine a particular data point's **percentile**, which is the **percent of the population less than the data point**.

Exercise #5: The average weight of full grown beef cows is 1470 pounds with a standard deviation of 230 pounds. If the weights are normally distributed, what is the percentile rank of a cow that weighs 1,750 pounds?

- (1) 89th (3) 49th
- (2) 76th (4) 35th

Your graphing calculator can also find these proportions or percent values. Each calculator's inputs and language will be slightly different, although many will do much of the work for you, even allowing you to **not think about the z-scores**.

Exercise #6: Given that the volume of soda in a 12 ounce bottle from a factory varies normally with a mean of 12.2 ounces and a standard deviation of 0.6 ounces, use your calculator to determine the probability that a bottle chosen at random would have a volume:

- (a) Greater than 13 ounces. (b) Less than 11 ounces (c) Between 11.5 and 12.5 ounces



Name: _____

Date: _____

THE NORMAL DISTRIBUTION AND Z-SCORES
COMMON CORE ALGEBRA II HOMEWORK

FLUENCY

1. A population has a mean of $\mu = 24.8$ and a standard deviation of $\sigma = 4.2$. For each of the following data values, calculate the z-value to the nearest hundredth. You do *not* need to read the Normal table.

(a) $x_i = 30$

(b) $x_i = 35$

(c) $x_i = 19$

(d) $x_i = 15.4$

(e) $x_i = 24.8$

(f) $x_i = 33.2$

2. A population has a mean of 102.8 and a standard deviation of 15.4. If a data point has a z-value of 1.87 then which of the following is the value of the data point?

(1) 28.8

(3) 131.6

(2) 86.7

(4) 152.3

APPLICATIONS

Get practice with both the Normal Distribution Table and your calculator when doing the following problems.

3. A recent study found that the mean amount spent by individuals on a music service website was normally distributed with a mean of \$384 with a standard deviation of \$48. Which of the following gives the proportion of the individuals that spend more than \$400?

(1) 0.43

(3) 0.12

(2) 0.74

(4) 0.37

4. The hold time experienced by people calling a government agency was found to be normally distributed with a mean of 12.4 minutes and a standard deviation of 4.3 minutes. Which percent below represents the percent of calls answered in less than 5 minutes?

(1) 4.3%

(3) 6.8%

(2) 5.3%

(4) 12.9%



5. The national average price per gallon for gasoline is normally distributed with a mean (currently) of \$2.34 per gallon with a standard deviation of \$0.26 per gallon. Which of the following represents the proportion of the gas prices that lie between \$2.00 and \$3.00?
- (1) 56% (3) 84%
(2) 72% (4) 90%
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6. If the average teacher salary in the United States is \$45,753 and salaries are normally distributed with a standard deviation of \$7890, would a salary of \$40,000 per year be in the lowest quintile of teacher salary? (Do a quick Internet search on the term quintile if you don't know what it means).
7. The average rent for a one bedroom apartment (in the Winter of 2015) in New York City is a whopping \$2801 per month with a standard deviation of \$920.
- (a) If rents are normally distributed, what percent of the apartments will be less than \$2,500 per month?
- (b) If rents are normally distributed, how realistic is it to believe you will be able to rent a one-bedroom in New York City for less than \$1,500 per month? Justify your answer.
- (c) A one-bedroom on the Upper East Side with a doorman and views of Central Park was listed at \$5,000 per month. How rare is this? Assume the rents are normally distributed.
- (d) Do you think the rents are normally distributed? Keep in mind the normal distribution is symmetric about its mean (looks the same on both sides). If it isn't symmetric, what does it look like?
8. A national math competition advances to the second round only the top 5% of all participants based on scores from a first round exam. Their scores are normally distributed with a mean of 76.2 and a standard deviation of 17.1. What score, to the nearest whole number, would be necessary to make it to the second round? To start, look at the table and see if you can determine the z-value that corresponds to the top 5%.



STANDARD NORMAL DISTRIBUTION BASED ON Z-VALUE

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0199	0.0239	0.0279	0.0319	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0753
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480	0.1517
0.4	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2088	0.2123	0.2157	0.2190	0.2224
0.6	0.2257	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2517	0.2549
0.7	0.2580	0.2611	0.2642	0.2673	0.2704	0.2734	0.2764	0.2794	0.2823	0.2852
0.8	0.2881	0.2910	0.2939	0.2967	0.2995	0.3023	0.3051	0.3078	0.3106	0.3133
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3365	0.3389
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3531	0.3554	0.3577	0.3599	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4429	0.4441
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4761	0.4767
2.0	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.4890
2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
2.4	0.4918	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
2.5	0.4938	0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.4960	0.4961	0.4962	0.4963	0.4964
2.7	0.4965	0.4966	0.4967	0.4968	0.4969	0.4970	0.4971	0.4972	0.4973	0.4974
2.8	0.4974	0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4979	0.4980	0.4981
2.9	0.4981	0.4982	0.4982	0.4983	0.4984	0.4984	0.4985	0.4985	0.4986	0.4986
3.0	0.4987	0.4987	0.4987	0.4988	0.4988	0.4989	0.4989	0.4989	0.4990	0.4990
3.1	0.4990	0.4991	0.4991	0.4991	0.4992	0.4992	0.4992	0.4992	0.4993	0.4993
3.2	0.4993	0.4993	0.4994	0.4994	0.4994	0.4994	0.4994	0.4995	0.4995	0.4995
3.3	0.4995	0.4995	0.4995	0.4996	0.4996	0.4996	0.4996	0.4996	0.4996	0.4997
3.4	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4998



