Name: ______________________________ Date: ______________

OTHER TYPES OF REGRESSION
COMMON CORE ALGEBRA II

Just as we fit data with a linear model we can also fit with all sorts of other mathematical models, depending on the context of the situation. In this lesson we will examine **exponential regression** and **sinusoidal regression**. Exponential regression is review from Common Core Algebra I, so we will start with that.

***Exercise* #1:** The population of Jamestown has been recorded for selected years since 2000. The table below gives these populations.

Year	2002	2004	2005	2007	2009
Population	5564	6121	6300	6812	7422

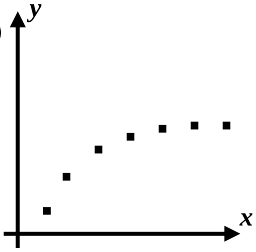
(a) Using your calculator, determine a best fit exponential equation, of the form $y = a \cdot b^x$, where x represents the number of years since 2000 and y represents the population. Round a to the nearest integer and b to the nearest *thousandth*.

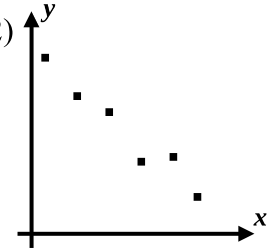
(b) Sketch a graph of the exponential function for the years 2000 to 2050. Label your window and your y-intercept.

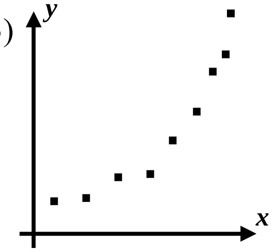
(c) By what percent does your exponential model predict the population is increasing per year? Explain.

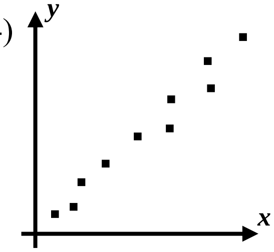
(d) Algebraically determine the number of years, to the nearest year, for the population to reach 20 thousand.

***Exercise* #2:** Which of the following scatter plots would be best fit with an exponential equation?

(1)


(2)


(3)


(4)


Sinusoidal, or trigonometric, regression is much more complicated than either linear or exponential. It should be used in situations that appear **periodic** in nature.

***Exercise* #3:** The temperature of a chemical reaction changes during the reaction. The temperature was measured every two minutes and the data is shown in the table below.

Time (min)	0	2	4	6	8	10	12	14	16	18	20
Temp $\left({}^\circ C\right)$	35.7	38.9	41.6	42.3	40.8	38.4	36.1	34.2	35.9	39.1	41

(a) Why does it seem like this data might be periodic? Create a quick scatter plot using your calculator to verify.

(b) Use your calculator to do a sine regression in the form $y = a\sin\left(bx + c\right) + d$. Round all parameters to the nearest tenth. Graph along with your data to informally assess the fit of the curve.

(c) According to this model, what is the range in temperatures the chemical reaction will include?

(d) According to this model, what is the time it takes for the reaction to complete one full cycle?

Graphing calculators vary. Many will require that if the period of the sinusoidal function is **unknown** then the data must have inputs that are separated by equal amounts (equal steps between *x*-values). On the other hand, there are many periodic phenomena that we want to model whose periods are known. In this case, we can enter data at irregular input intervals.

***Exercise* #4:** The maximum amount of daylight that hits a spot on Earth is a function of the day of the year. Taking $x = 0$ to be January 1st, daylight, in hours, was measured for 12 different days. The measurement was the number of possible hours of sun from sunrise to sunset.

Day	0	34	68	98	118	134	171	203	274	321	346
Daylight Hours	9.0	9.9	11.5	13.1	14.0	14.6	15.2	14.8	13.1	11.5	9.5

(a) What is the natural period of this data set?

(b) Use your calculator with the period from (a) to find an equation of the form $y = a\sin\left(bx + c\right) + d$ that fits this data, then examine the graph of the equation on the scatter plot. How good is the fit?

(c) What is the maximum amount of daylight hours predicted by the model? Show your calculation.

Name: ___________________________________ Date: _________________

OTHER TYPES OF CORRELATION
COMMON CORE ALGEBRA II HOMEWORK

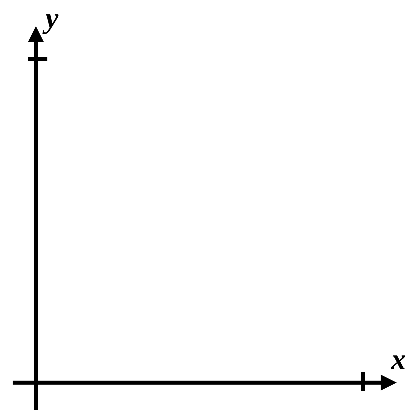
APPLICATIONS

1. Rabbits were accidently introduced to an island where their population is growing rapidly. Biologists studying the rabbits have periodically recorded their population since they were introduced to the island. The data they took is shown below.

Years Since Introduction, x	2	5	7	11	15
Population of Rabbits, y	75	100	112	205	290

(a) Determine an exponential regression equation, in the form $y = a \cdot b^x$, that models this data. Round a to the *tenth* and b to the *hundredth*.

(b) Sketch a graph of the rabbit population below on the axes provided for $0 \le x \le 20$. Label your graphing window and your y-intercept.



(c) Based on your model in part (a), by what percent is the rabbit population growing each year?

(d) Graphically determine, to the nearest *tenth* of a year, when the rabbit population will reach 350.

2. The infiltration rate of a soil is the number of inches or water per hour it can absorb. Hydrologists studied one particular soil and found its infiltration rate decreases exponentially as a rainfall continues.

Time, t (hours)	0	1.5	3.0	4.5	6.0
Infiltration Rate, I (inches per hour)	5.3	3.1	2.4	1.6	0.7

Create an exponential model that best fits this data set. Round parameters to the nearest *hundredth*. Use your model to algebraically determine the time until the rate reaches 0.25 inches per hour. Round your answer to the nearest *tenth* of an hour. Use a logarithm in the process of your algebraic solution.

3. The soil's temperature beneath the ground varies in a periodic manner. A temperature probe was left 3 feet underground and recorded the temperature as a function of the number of days since January 1st ($x=0$). The temperatures for 14 days throughout the year are shown below.

Day	5	36	57	94	127	153	192
Temp ($^\circ$F)	41	37	36	40	48	64	68
Day	226	241	262	289	305	337	356
Temp ($^\circ$F)	66	61	58	49	44	42	40

(a) Find a best fit sinusoidal function for this data set in the form $y=a\sin(bx+c)+d$. Round all parameters to the nearest *hundredth*. Recall that some calculators require that you input the period on this correlation (365 days).

(b) Based on your model from (a) what are the highest and lowest temperature reached in the soil?

(c) What is the average soil temperature?

(d) If the root of a particular plant species will only thrive when the soil temperature is above 50 $^\circ$F, graphically determine the interval of days over which the plant will thrive.

4. The rise and fall of the tides at a beach is recorded at regular intervals. Their period is almost 24 hours, but not exactly. The depth of a tidal marsh was measured over 3-hour time interval and the data is shown below.

Hours (since midnight)	0	3	6	9	12	15	18	21	24
Depth (ft)	5.5	8.0	10.5	11.7	10.8	8.4	5.8	4.3	4.9

Find a sinusoidal model for this data using your calculator. Place it in $y=a\sin(bx+c)+d$ form. Round all coefficients to the nearest *thousandth* (3 decimal places).

According to your model, what is the period of the tides in hours? Recall that $b\cdot P=2\pi$.