Name: ____________________________________ Date: __________________

CONDITIONAL PROBABILITY
COMMON CORE ALGEBRA II

When the probability of one event occurring changes depending on other events occurring then we say that there is a **conditional probability**. The language and symbolism of conditional probability can be a bit confusing, but the idea is fairly straightforward and can be developed with two-way frequency charts.

***Exercise* #1:** Let's revisit a two-way frequency chart we saw in the last lesson. In this study, 52 graduating seniors were surveyed as to their post-graduation plans and then the results were sorted by gender.

Let the following letters stand for the following events.

M = Male F = Female

C = Going to College N = Not going to college

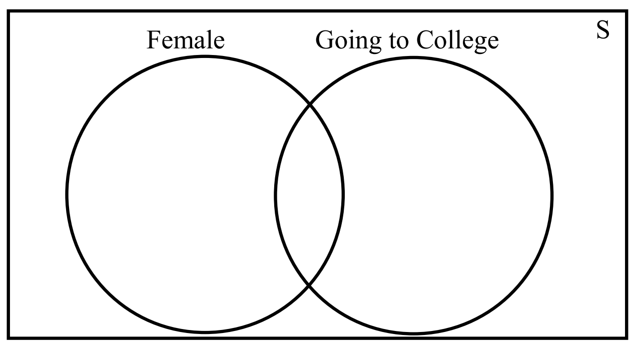
	Gender		
	Male	Female	Total
Going to College	16	13	29
Not Going to College	14	9	23
Total	30	22	52

If a person was picked at random, find the probability that the person was

(a) a female, i.e. $P(\mathrm{F})$

(b) going to college $P(\mathrm{C})$

(c) going to college **given** they are female, i.e. $P(\mathrm{C} \mid \mathrm{F})$. Draw a Venn diagram below to help justify the ratio that you give as the probability.

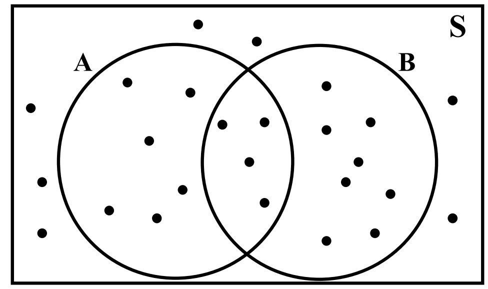


(d) Which is more likely, that a person picked at random will be going to college, given they are a male, i.e. $P(\mathrm{C} \mid \mathrm{M})$, or that a person will be male, given they are going to college, i.e. $P(\mathrm{M} \mid \mathrm{C})$. Show that calculations for both.

$P(\mathrm{C} \mid \mathrm{M})$ $P(\mathrm{M} \mid \mathrm{C})$

We can generalize this process to calculate these conditional probabilities based on counts and a way to calculate these probabilities based on other probabilities.

***Exercise* #2:** In the generic Venn diagram shown to the right. Each dot represents an equally likely outcome of the sample space. Some of these fall only into event A, some only into event B, some in both events and some in neither.



(a) Consider the probability of A occurring given that B has occurred. Give a formula for this probability based on counting the number of elements in each set and their intersection.

$P(\text{A} \mid \text{B}) =$

(b) Divide both of the numerator and denominator in (a) by the number of total elements in the sample space. Then rewrite the formula in (a) in terms of probabilities instead of counts.

$P(\text{A} \mid \text{B}) =$

It's great when we can count elements that lie in events and their intersection, but sometimes we cannot. For example, let's revisit a relative frequency table that we saw in a previous homework.

***Exercise* #3:** A survey was taken to examine the relationship between hair color and eye color. The chart below shows the proportion of the people surveyed who fell into each category. If a person was picked at random, find each of the following conditional probabilities. Show the calculation you used.

		Hair Color			
		Black	Blond	Red	Total
Eye Color	Blue	0.15	0.20	0.05	0.40
	Brown	0.25	0.10	0.00	0.35
	Green	0.05	0.05	0.15	0.25
	Total	0.45	0.35	0.20	1.00

(a) Find the probability the person picked had brown eyes given they had blond hair.

$P(\text{brown eyes} \mid \text{blond hair})$

(b) Find the probability the person had red hair given they had green eyes.

$P(\text{red hair} \mid \text{green eyes})$

(c) Does having red hair seem have some **dependence** on having green eyes? How can you tell or quantify this dependence?

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CONDITIONAL PROBABILITY
COMMON CORE ALGEBRA II HOMEWORK

FLUENCY

1. Given that $P(\text{B} \mid \text{A})$ means the probability of event B occurring given that event A will occur or has occurred, which of the following correctly calculates this probability?

(1) $\dfrac{P(\text{B})}{P(\text{A})}$ (3) $\dfrac{P(\text{A})}{P(\text{B})}$

(2) $\dfrac{P(\text{A and B})}{P(\text{B})}$ (4) $\dfrac{P(\text{A and B})}{P(\text{A})}$

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APPLICATIONS

2. Of the 650 juniors at Arlington High School, 468 are enrolled in Algebra II, 292 are enrolled in Physics, and 180 are taking both courses at the same time. If one of the 650 juniors was picked at random, what is the probability they are taking Physics, if we know they are in Algebra II?

(1) 0.38 (3) 0.45

(2) 0.62 (4) 0.58

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3. Historically, a given day at the beginning of March in upstate New York has a 18% chance of snow and a 12% chance of rain. If there is a 4% chance it will rain and snow on a day, then calculate each of the following:

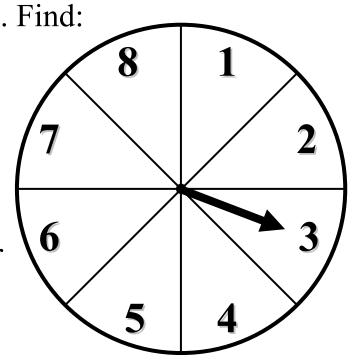
(a) the probability it will rain given that it is snowing, i.e.

$P(\text{rain} \mid \text{snow})$

(b) the probability it will snow given that it is raining, i.e.

$P(\text{snow} \mid \text{rain})$

4. A spinner is spun around a circle that is divided up into eight equally sized sectors. Find:



(a) $P(\text{perfect square} \mid \text{even})$ (b) $P(\text{odd} \mid \text{prime})$

(c) What is more likely: getting a multiple of four given we spun an even or getting an odd, given we spun a number greater than 2? Support your answer.

5. A survey was done of commuters in three major cities about how they primarily got to work. The results are shown in the frequency table below. Answer the following conditional probability questions.

	Car	Train	Walk	Total
New York	.05	.25	.10	.40
Los Angeles	.18	.12	.05	.35
Chicago	.08	.14	.03	.25
Total	.31	.51	.18	1.00

(a) What is the probability that a person picked at random would take a train to work given that they live in Los Angeles.

$P(\text{train} \mid \text{LA})$

(b) What is the probability that a person picked at random would live in New York given that they drive a car to work.

$P(\text{NYC} \mid \text{Car})$

(c) Is it more likely that a person who takes a train to work lives in Chicago or more likely that a person who lives in Chicago will take a train to work. Support your work using conditional probabilities.

REASONING

6. The formula for conditional probability is: $P(\text{B} \mid \text{A}) = \dfrac{P(\text{A and B})}{P(\text{A})}$. Solve this formula for $P(\text{A and B})$.

7. We say **two events**, A and B, **are independent** if the following is true:

$$P(\text{B} \mid \text{A}) = P(\text{B}) \text{ and likewise } P(\text{A} \mid \text{B}) = P(\text{A})$$

Interpret what the definition of **independent events** means in your own words.