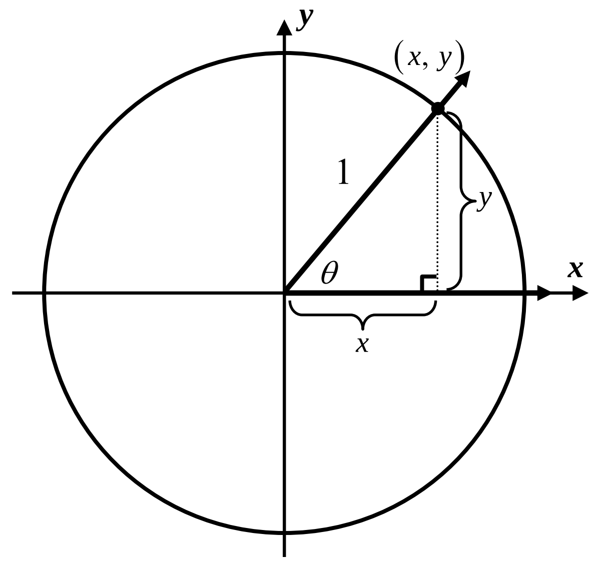
Name: ____________________________________ Date: _________________

THE DEFINITION OF THE SINE AND COSINE FUNCTIONS
COMMON CORE ALGEBRA II

The sine and cosine functions form the basis of trigonometry. We would like to define them so that their definition is consistent with what you already are familiar with concerning right triangle trigonometry. Recall from Common Core Geometry that in a right triangle the sine and cosine ratios were defined as:

$$\sin A = \frac{\text{side length opposite of } A}{\text{length of the hypotenuse}} \quad \text{and} \quad \cos A = \frac{\text{side length adjacent to } A}{\text{length of the hypotenuse}}$$

***Exercise* #1:** Consider the **unit circle** shown below with an angle, θ, drawn in standard position.



(a) Given the right triangle shown, find an expression for $\sin(\theta)$.

(b) Given the right triangle shown, find an expression for $\cos(\theta)$.

We thus define the sine and cosine functions by using the coordinates on the unit circle. They are the first functions that are **geometrically defined** as they are based on the geometry of a circle (circular functions).

THE DEFINITION OF THE SINE AND COSINE FUNCTIONS

For an angle in standard position whose terminal ray passes through the point (x, y) on the unit circle:

$\sin(\theta) =$ the y-coordinate and $\cos(\theta) =$ the x-coordinate

The above definition is **unquestionably the most important fact to memorize** concerning trigonometry. We can now use this along with our work on the unit circle to determine certain **exact** values of cosine and sine.

***Exercise* #2:** Using the unit circle diagram, determine each of the following values.

(a) $\sin(30^\circ) =$ (b) $\sin(240^\circ) =$ (c) $\cos(90^\circ) =$ (d) $\cos(180^\circ) =$

(e) $\sin(90^\circ) =$ (f) $\sin(135^\circ) =$ (g) $\cos(150^\circ) =$ (h) $\cos(0^\circ) =$

***Exercise* #3:** The terminal ray of an angle, α, drawn in standard position passes through the point $(-0.6, 0.8)$, which lies on the unit circle. Which of the following gives the value of $\sin(\alpha)$?

(1) 1.2 (3) -0.6

(2) 0.8 (4) 0.2 ________

It is important to be able to determine the sign (positive or negative) of each of the two basic trigonometric functions for an angle whose terminal ray lies in a given quadrant. The next exercise illustrates this process.

***Exercise* #4:** For each quadrant below, determine if the sine and cosine of an angle whose terminal ray falls in the quadrant is positive $(+)$ or negative $(-)$.

	I	II	III	IV
$\sin(\theta)$				
$\cos(\theta)$				

Since each point on the unit circle must satisfy the equation $x^2 + y^2 = 1$, we can now state what is known as the **Pythagorean Identity**.

THE PYTHAGOREAN IDENTITY

For any angle, θ, $(\cos\theta)^2 + (\sin\theta)^2 = 1$

***Exercise* #5:** An angle, α, has a terminal ray that falls in the second quadrant. If it is known that $\sin(\alpha) = \frac{3}{5}$, determine the value of $\cos(\alpha)$.

***Exercise* #6:** An angle, θ, has a terminal ray that falls in the first quadrant and $\cos(\theta) = \frac{1}{3}$. Determine the value of $\sin(\theta)$ in simplest radical form.

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THE DEFINITION OF THE SINE AND COSINE FUNCTIONS
COMMON CORE ALGEBRA II HOMEWORK

FLUENCY

1. Which of the following is the value of $\sin\left(60^\circ\right)$?

(1) $\frac{\sqrt{2}}{2}$ (3) $\frac{\sqrt{3}}{2}$

(2) $\frac{1}{2}$ (4) $\frac{2}{3}$

____

2. Written in exact form, $\cos\left(135^\circ\right)=?$

(1) $-\frac{1}{2}$ (3) $-\frac{\sqrt{3}}{2}$

(2) $-\frac{\sqrt{2}}{2}$ (4) $-\frac{\pi}{4}$

____

3. Which of the following is not equal to $\sin\left(270^\circ\right)$?

(1) $\cos\left(180^\circ\right)$ (3) $-\sin\left(90^\circ\right)$

(2) $-\cos\left(0^\circ\right)$ (4) $\sin\left(360^\circ\right)$

____

4. The terminal ray of an angle drawn in standard position passes through the point $\left(0.28,-0.96\right)$, which lies on the unit circle. Which of the following represents the sine of this angle?

(1) –0.96 (3) 0.28

(2) –0.68 (4) –0.29

____

5. The point $A\left(-5,12\right)$ lies on the circle whose equation is $x^2+y^2=169$. Which of the following would represent the cosine of an angle drawn in standard position whose terminal rays passes through A?

(1) –5 (3) $-\frac{5}{13}$

(2) $-\frac{5}{12}$ (4) 12

____

6. Which of the following values cannot be the sine of an angle? Hint, think about the range of y-values on the unit circle.

(1) $\frac{7}{13}$ (3) $-\frac{3}{2}$

(2) $-\frac{\sqrt{5}}{3}$ (4) $\frac{\sqrt{11}}{4}$ __________

7. For an angle drawn in standard position, it is known that its cosine is negative and its sine is positive. The terminal ray of this angle must terminate in which quadrant?

(1) I (3) III

(2) II (4) IV __________

8. If both the sine and cosine of an angle are less than zero, then when drawn in standard position in which quadrant would the terminal ray fall?

(1) I (3) III

(2) II (4) IV __________

9. Which of the following has a cosine that is different from $\sin\left(30^{\circ}\right)$?

(1) 60° (3) -60°

(2) -300° (4) 120° __________

10. When drawn in standard position, an angle α has a terminal ray that lies in the second quadrant and whose sine is equal to $\frac{9}{41}$. Find the cosine of α in rational form (as a fraction).

11. If the terminal ray of β lies in the fourth quadrant and $\sin(\beta)=-\frac{\sqrt{3}}{3}$ determine $\cos(\beta)$ in simplest form.