Name: ____________________________________ Date: __________________

POLYNOMIAL IDENTITIES
COMMON CORE ALGEBRA II

Polynomials are expressions consisting of addition and subtraction of variables and coefficients all raised to non-negative, integer powers. As in the last few lessons, there can be a single variable or multiple variables.

***Exercise* #1:** Which of the following is *not* a polynomial expression? Explain why your choice fails to be a polynomial.

(1) $x^3+2x^2y+y^3$

(2) x^7+y^7

(3) $x^{-4}y^2+2xy^{1/2}$

(4) x^2-y^2

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Because polynomials consist of basic operations on variables, they can be manipulated using the associative, commutative, and distributive properties (as you have done many times). These operations can result in what are known as **polynomial identities**. An identity is defined more broadly below:

IDENTITIES

An **identity** is an **equation** that is **true** for all values of the replacement variable or variables.

***Exercise* #2:** One identity that you should be familiar with is $x^2-y^2=(x-y)(x+y)$.

(a) Test this identity with the pair $x=10$ and $y=3$.

(b) Prove this identity by manipulating the right side of the equation.

(c) Use this identity to evaluate the difference 50^2-49^2.

(d) Use this identify to simplify and then evaluate the product $(51)(49)$.

***Exercise* #3:** Prove the identity $(a+b)^2=a^2+2ab+b^2$ and use it to evaluate 35^2.

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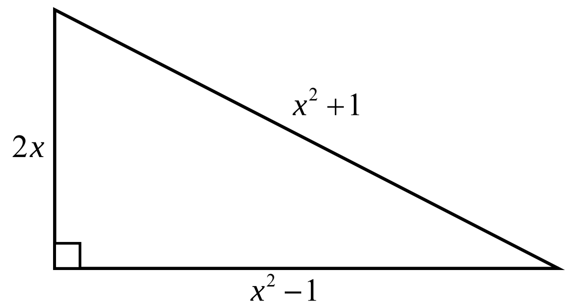
Sometimes identities can have geometric connections as well as algebraic. The Pythagorean Theorem gives us ample identities.

***Exercise* #4:** A right triangle is shown below whose sides are $2x, x^2-1,$ and x^2+1.



(a) Show that these will be the side lengths of a right triangle as long as $x>1$, i.e. show that

$$(2x)^2+(x^2-1)^2=(x^2+1)^2$$

is an identity.

(b) Based on our work from (a) and on the triangle shown, explain why any even integer (other than 2) must be part of a Pythagorean triple, i.e. a set of 3 **integers** that could be the sides of a right triangle. Generate a Pythagorean triple that has 10 in it and a separate one that has 14 in it.

***Exercise* #5:** Consider the polynomial identity $(a+b)^3=a^3+3a^2b+3ab^2+b^3$.

(a) Prove this identity by expanding the left-hand side of the equation.

(b) Use your calculator to find the value of 11^3 then use the identity to show the same result. Carefully consider your choice of *a* and *b*.

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POLYNOMIAL IDENTITIES
COMMON CORE ALGEBRA II HOMEWORK

FLUENCY

1. One of the two expressions below is an identity and one of them is not. Determine which is an identity by testing the truth value of the equation for various values of *x*. Show the values of *x* that you test. Remember, an identity will be true for **every value of *x*.**

Equation #1: $(x+1)^2 = 4x+4$ Equation #2: $(x+2)^2 = x^2+4x+4$

2. Which of the following equations represents an identity?

(1) $2x+1=3x+4$ (3) $4x-3=2(2x+7)$

(2) $6x+3=2x+10$ (4) $4(5x+2)=20x+8$

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3. One of the more useful identities that students almost inherently learn is:

$$(x+c)(x+d)=x^2+(c+d)x+cd$$

(a) Prove this identity. You may choose to algebraically manipulate one or both sides of the equation to justify the equivalence.

(b) This identity allows you to multiply common binomials very quickly. Find the following products in simplest trinomial form.

(i) $(x+3)(x+7)$ (ii) $(x-7)(x-2)$ (iii) $(x+10)(x-3)$

4. You should be well aware of the difference of perfect squares, i.e. $x^2-y^2=(x-y)(x+y)$. But there is also an identity for the difference of perfect cubes:

$$x^3-y^3=(x-y)(x^2+xy+y^2)$$

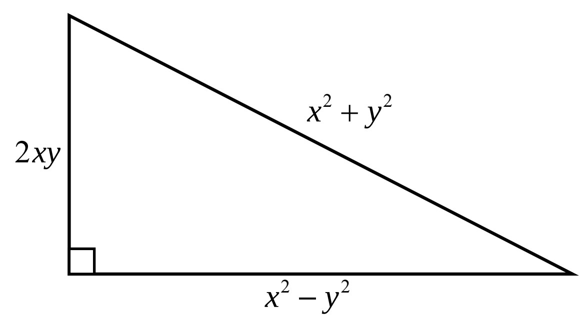
(a) Prove this identity by expanding the product on the right-hand side of the equation.

(b) Use the identity to find the value of 10^3-9^3 without the use of your calculator. Show the steps in your calculation. Then, verify with your calculator.

APPLICATIONS

5. Another famous identity that can be used to generate Pythagorean Triples is shown below:

$$(x^2-y^2)^2+(2xy)^2=(x^2+y^2)^2$$



The complicated sides are shown on the diagram.

(a) Prove this identity by expanding the products on both sides of the equation.

(b) Generate the sides of the right triangle if $x=4$ and $y=1$. Show that these sides satisfy the Pythagorean Theorem.

(c) **Reasoning:** This relationship leads to the conclusion that there is *no* Pythagorean triple of this form that contains the integer 2. Why?

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