

**Aim:** How do we “reduce” trig function values to trig functions of positive acute angles?

**I. Do Now:**

1. Find the exact value of each:

(a)  $\sin \frac{\pi}{3}$

(b)  $\sin \frac{2\pi}{3}$

(c)  $\sin \frac{4\pi}{3}$

(d)  $\sin \frac{5\pi}{3}$

(e)  $\cos 45^\circ$

(f)  $\cos 135^\circ$

(g)  $\cos 225^\circ$

(h)  $\cos 315^\circ$

2. Find the exact values of all six trigonometric functions of  $\theta$  if the point  $(-8, -11)$  is on the terminal side of  $\theta$ .

**II. “Reducing” Trig Functions of Any Angle to a Trig Function of a Positive Acute Angle**

Due to the circular nature of trig functions (they are sometimes referred to as circular functions), the value of a trig function at a given angle is always the same as its value at that angle’s reference angle, except when there is a variation in sign. Using A-S-T-C, we can “reduce” any trig function value to a trig function of its reference angle. (Note: Using cofunctions, any trig function can be further “reduced to a function of a positive acute angle less than  $45^\circ$ .”)

*Examples:*

$$\sin 240^\circ = \ominus \sin 60^\circ$$

$$= -\sin 60^\circ$$

$$= -\frac{\sqrt{3}}{2}$$

$$\cos 300^\circ = \oplus \cos 60^\circ$$

$$= \cos 60^\circ$$

$$= \frac{1}{2}$$

**III. Express as a function of a positive acute angle and find the exact value without using a calculator:**

1.  $\sin 120^\circ$

7.  $\cot \frac{2\pi}{3}$

2.  $\cos 210^\circ$

8.  $\cos \frac{5\pi}{3}$

3.  $\tan 135^\circ$

9.  $\sin \frac{17\pi}{3}$

4.  $\sec \frac{4\pi}{3}$

10.  $\csc \frac{7\pi}{6}$

5.  $\tan \frac{5\pi}{4}$

11.  $\tan\left(-\frac{13\pi}{6}\right)$

6.  $\csc \frac{5\pi}{6}$

12.  $\cot\left(-\frac{15\pi}{4}\right)$

**IV. Practice with Quadrantal Angles: Evaluate:**

13.  $\sin 90^\circ$

17.  $\cot \frac{\pi}{2}$

14.  $\cos 180^\circ$

18.  $\sec 7\pi$

15.  $\tan 360^\circ$

19.  $\sin\left(-\frac{11\pi}{2}\right)$

16.  $\csc 270^\circ$

20.  $\tan 3\pi$

**V. More Practice:**

Express the given function as a function of a positive acute angle and, if possible, find the exact value:

21.  $\csc 1,200^\circ$

25.  $\sec \frac{50\pi}{3}$

22.  $\tan 345^\circ$

26.  $\csc \frac{13\pi}{7}$

23.  $\cos 208^\circ$

27.  $\sin\left(-\frac{147\pi}{4}\right)$

24.  $\sin(-250^\circ)$

28.  $\cos \frac{19\pi}{5}$

Find the exact value of the given expression:

29.  $\left(\sec \frac{2\pi}{3}\right)\left(\sin \frac{2\pi}{3}\right)$

30.  $\csc \frac{\pi}{4} + \cot \frac{3\pi}{4} + \sec 5\pi$

31.  $\tan \frac{\pi}{3} + \cot \frac{5\pi}{6}$