Name: ______________________ **Precalculus Unit 10, Lesson 18**

Aim: How do we find the *n*th roots of a complex number? (Day 2)

I. Do Now:

1. Find all four fourth roots of 1 and represent them graphically on the complex plane.

> The *n*th Roots of Unity
> A complex number $v = a + bi$ is an nth root of z if $v^n = z$.
> If $z = 1$, then v is called an nth root of unity.
> Since $z = 1 = 1 + 0i$ is equivalent to $\cos 0 + i\sin 0$ in trigonometric form, the nth roots of unity are $\cos\frac{2\pi k}{n} + i\sin\frac{2\pi k}{n}$ for $k = 0, 1, 2, \ldots, n-1$.

II. *n*th Roots of a Complex Number

For a positive integer n, the complex number $z = r(\cos\theta + i\sin\theta)$ has exactly n distinct nth roots given by $\sqrt[n]{r}\left[\cos\left(\frac{\theta+2\pi k}{n}\right) + i\sin\left(\frac{\theta+2\pi k}{n}\right)\right]$, where $k = 0, 1, 2, \ldots, n-1$.

There are always n nth roots of a number, if we include all complex roots. If all roots of a number are graphed on the complex plane, they will be located $360°/n$ (or $2\pi/n$ radians) apart.

To find all of them quickly:

1) Convert the number z that you wish to find roots of to trigonometric ($r\operatorname{cis}\theta$) form.
2) Find the 1st nth root of z by taking the nth root of the modulus and by multiplying the argument by $1/n$.
3) Find all other roots by adding multiples of $2\pi/n$ (or $360°/n$) to the argument of the 1st root.

III. Applications

2. Solve the equation $x^6 = 1$.
(i.e., find the sixth roots of unity)

3. Find the fourth roots of $3 + 3i\sqrt{3}$.
Leave your answers in trigonometric form.

4. Find the sixth roots of 64
in $a + bi$ form.

5. Find the fifth roots of unity.
Leave your answers in trigonometric form.

6. Solve the equation $x^5 + 243 = 0$
for all complex roots.
Leave your answers in trigonometric form.

7. Solve the equation $x^3 = -4\sqrt{2} - 4i\sqrt{2}$
for all complex roots.
Leave your answers in trigonometric form.

> HW42
> • Read pages 506 – 508.
> • p. 510: 50, 55, 80, 87 (hint: raise it to the 6th power), 95, 106, 112, 113.