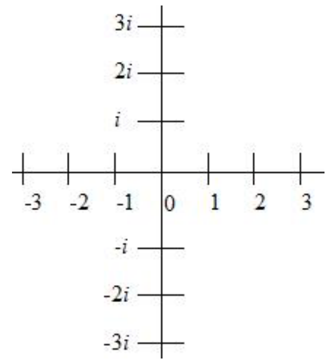
Name: ______________________ **Precalculus Unit 10, Lesson 17**

Aim: How do we find the *n*th roots of a complex number?

I. Do Now:

1. (a) Plot i, i^2, i^3, i^4, and i^5 on the complex plane on the right.

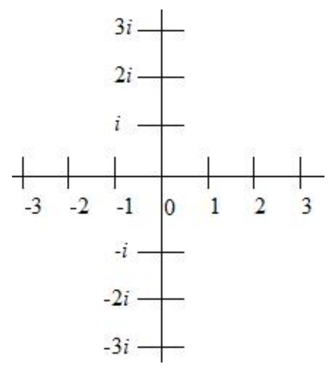


   (b) Write each number above in trigonometric form:
   $i =$
   $i^2 =$
   $i^3 =$
   $i^4 =$
   $i^5 =$

   (c) What happens to r and θ as the power of i increases?

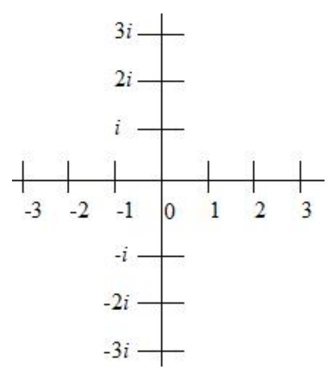
   (d) Where would $\sqrt{i} = i^{\frac{1}{2}}$ be located?

2. Solve for *all* complex roots of $x^3 - 1 = 0$ and plot them on the complex plane on the right.

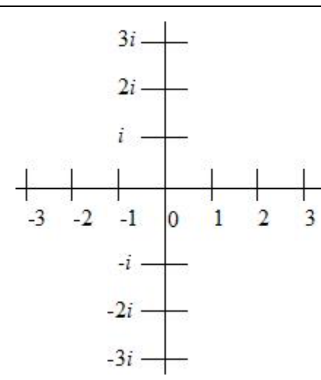


II. Development:

3. Find all three cube roots of -1.
   Let $x^3 = -1 \Rightarrow x^3 + 1 = 0$ and solve by factoring the sum of cubes (You will need to use the Quadratic Formula to find two of the roots). Then, graph the solutions on the complex plane on the right.



4. Find all fourth roots of 16.
   Let $x^4 = 16$ and solve by factoring. Graph the solutions on the complex plane on the right.



III. *n*th Roots of a Complex Number

There are always n nth roots of a number, if we include all complex roots. If all roots of a number are graphed on the complex plane, they will be located ________ degrees (or _______ radians) apart.

To find all of them quickly:

1) Convert the number z that you wish to find roots of to trigonometric ($r\,\text{cis}\,\theta$) form.
2) Find the 1st root of z by taking the nth root of the modulus and by multiplying the argument by $1/n$.
3) Find all other roots by adding multiples of $2\pi/n$ (or $360°/n$) to the argument of the 1st root.

[Note: There is a formula describing this process on page 507 of your textbook.]

IV. Applications

5. Find all cube roots of 8.

6. Find both square roots of $16\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)$ and convert them to $a + bi$ form.