Related Rate Problems Sheet 1

These formulas will be helpful in solving the problems below:

$$\text{Volume of a cone} = \tfrac{1}{3}\pi r^2 h$$

$$\text{Volume of sphere} = \tfrac{4}{3}\pi r^3$$

$$\text{Volume of cylinder} = \pi r^2 h$$

$$\text{Surface area of sphere} = 4\pi r^2$$

1)   Assume that oil spilled from a ruptured tanker spreads in a circular pattern whose radius increases at a constant rate of 2 feet per second. How fast is the area of the spill increasing when the radius of the spill is 60 feet?

2)   Air is pumped into a spherical balloon at a rate of 4.5 cubic inches per minute. Find the rate of change of the radius of the balloon when the radius is 2 inches.

3)   Two cars start from the same place and time. One car travels north at 25 mph and the other car travels at 60 mph going east. How fast is the distance between the cars increasing at the end of 5 hours?

4)   A 15 ft ladder is leaning against a wall. The foot of the ladder is slipping away from the wall at a rate of ½ foot every second. At what rate is the top of the ladder falling at the instant that the foot of the ladder is 9 feet from the wall?

5)   A rocket is rising vertically at 850 ft per second. A ground TV camera 3000 ft away is filming the launch. How fast is the distance between the camera and the rocket changing at the instant the rocket reaches an elevation of 4000 ft?

6)   A baseball diamond is a square whose sides are 90 ft long. Suppose that the player is running from second base and has a speed of 30 ft per second at the instant when he is 20 ft from third base. At what rate is the player's distance from home plate changing at that instant?

7)   An airplane is flying in still air with an air speed of 240 miles per hour. It is climbing at an angle of 22°. Find the rate at which it is gaining altitude.

8) A rocket is rising vertically at 880 ft per second. A ground-based camera 3000 ft away is filming the launch. How fast must the angle of elevation of the camera change at the instant when the rocket is 4000 ft high if the camera is to keep the rocket in sight?

9) Water is running out of a conical funnel at a rate of 1 cubic inch per second. If the radius of the base of the funnel is 4 inches and the altitude is 8 inches, find the rate at which the water level is dropping when it is 2 inches from the top.

10) How fast does the water level drop when a cylindrical tank of radius 20 cm is drained at a rate of 300 cubic centimeters per second?

11) When the price of a certain commodity is p dollars per unit, the manufacturer is willing to supply x hundred units where $3p^2 - x^2 = 12$. How fast is the supply changing when the price is $4 per unit and is increasing at a rate of 87¢ per month?

12) A trough is 5 ft long and its vertical cross-sections are isosceles triangles with base 2 ft and height 3 feet. Water is siphoned out at a rate of 2 cubic feet per minute.

   a. Find the volume of the water when the trough is full.

   b. Find the rate of change with respect to the depth of the trough at the instant when the trough is ½ full by volume.

13) A circle is inscribed in a square. The circumference of the circle increases at a constant rate of 6 inches per second. As the circle expands the square expands to the condition of tangency.

   a. Find the rate at which the perimeter of the square is increasing.

   b. Find the rate of increase in the area between the inscribed circle and the square when the area of the circle reaches 25π square inches.

14) The voltage, V, in an electrical circuit is related to the current, I, and the resistance, R, by the equation $V = IR$. The current is decreasing at -4 amps/sec as the resistance increases at 20 ohms/sec. How fast is the voltage changing when the voltage is 100 volts and the current is 20 amps?