

Properties of Limits

Let \lim stand for one of the limits $\lim_{x \rightarrow a}$, $\lim_{x \rightarrow a^-}$, $\lim_{x \rightarrow a^+}$, $\lim_{x \rightarrow +\infty}$, or $\lim_{x \rightarrow -\infty}$.

Assume that $\lim f(x)$ and $\lim g(x)$ both exist and let c be any constant.

1. The limit of a sum is the sum of the limits.

$$\lim [f(x) + g(x)] = \lim f(x) + \lim g(x)$$

2. The limit of a difference is the difference of the limits.

$$\lim [f(x) - g(x)] = \lim f(x) - \lim g(x)$$

3. A constant factor can be moved through a limit sign.

$$\lim [c \cdot f(x)] = c \cdot \lim f(x)$$

4. The limit of a product is the product of the limits.

$$\lim [f(x) \cdot g(x)] = \lim f(x) \cdot \lim g(x)$$

5. The limit of a quotient is the quotient of the limits (provided that the limit of the denominator is not zero).

$$\lim \left[\frac{f(x)}{g(x)} \right] = \frac{\lim f(x)}{\lim g(x)} \quad \text{provided that } \lim g(x) \neq 0$$

6. The limit of a power is the power of the limit.

$$\lim [f(x)]^n = [\lim f(x)]^n$$

7. The limit of an n th root is the n th root of the limit.

$$\lim \sqrt[n]{f(x)} = \sqrt[n]{\lim f(x)} \quad \text{provided that } \lim f(x) \geq 0 \text{ if } n \text{ is even}$$