MA1 – Optimization Problems Sheet 2

1. Find two non-negative numbers whose sum is 108 and the product of one with the cube of the other is a maximum.

2. A farmer wants a rectangular corral for his horses. He has 1200 meters of fencing. What are the dimensions which will give the largest possible corral?

3. Find the point in the first quadrant on the graph of $x^2 - y^2 = 16$ which is closest to the point (0,6).

4. Find the dimensions of a rectangle of largest area that can be inscribed in a circle of radius 4.

5. A poster is to contain a printed area of 150 square inches. There is to be a 3 inch margin on the top and bottom, and a 2 inch margin on each side. What are the dimensions of the smallest poster that can be used?

6. A man in a boat is 24 miles from a straight shore line, and wishes to reach a point 20 miles further down the shore line. He can travel 5 mph by boat and 13 mph on land. At what point should he land the boat in order to minimize the time required to reach his destination?

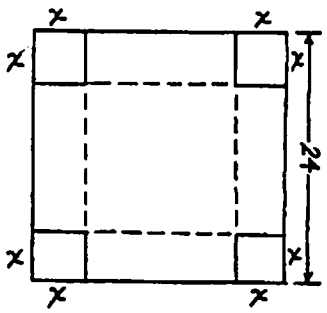
7. Find the dimensions of a right circular cylinder of maximum volume which can be inscribed in a sphere of radius 12. (Volume of a sphere: $V = \frac{4}{3}\pi r^3$; Volume of a cylinder: $V = \pi r^2 h$)

8. A tank with a rectangular base and rectangular sides is to be open at the top. It is to be constructed so that its width is 4 meters, and its volume is 36 cubic meters. If building the tank costs \$10 per square meter for the base, and \$5 per square meter for the sides, what is the cost of the least expensive tank?

9. A closed box with a square base is to contain 252 cubic feet. A bottom costs \$5 per square foot, the top costs \$2 per square foot, and the sides cost \$3 per square foot. Find the dimensions of the box that will minimize the cost.

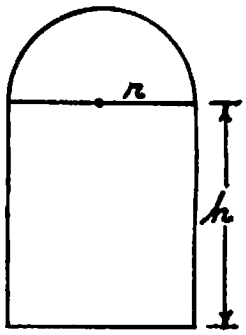
10. A closed cylindrical can is to have a volume of 1000 cm^3. Find the dimensions of the can that will minimize the amount of tin required to construct it.

11. An open rectangular box is to be constructed from a square sheet of metal of side 24 inches by cutting out square corners and folding up the sides. What size square corners should be cut out in order to maximize the volume of the box?

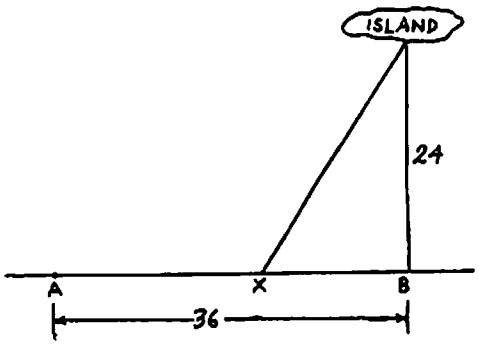


12. A wire 20 cm long is cut into 2 pieces. One piece is bent to form a circle, and the other *piece is bent to* form a square. Where should the cut be made so that the sum of the areas of the circle and square is maximized?

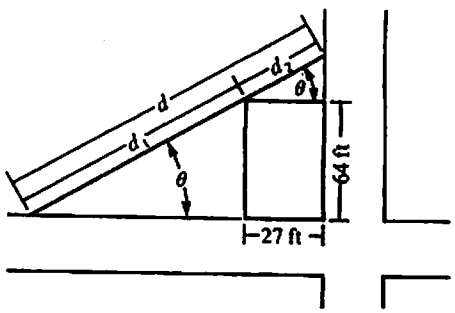
13. A Norman window is one in the shape of a rectangle topped by a semicircular region. If the perimeter of such a window is to be 12 feet, find the dimensions of the window which will maximize its area.



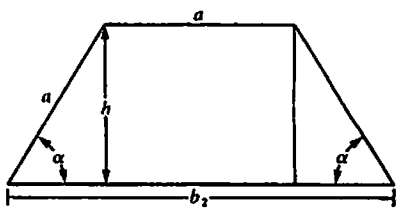
14. A company ships from a mainland plant (point A) to an island that is 24 miles off shore from point B. Point B is 36 miles from point A. The company wants to build a shipping station at point X, somewhere along the shore where the goods will be transferred from a truck to a boat. It costs \$1.00 per ton per mile to ship by truck, and \$1.25 per ton per mile to ship by boat. Where should the shipping station be built in order for the company to minimize its shipping costs?



15. A rectangular building is located on an intersection of two streets, as shown in the figure. The city wants to build a straight-line sidewalk from one street to the other, so that the sidewalk just touches the corner of the building. Find the angle θ which will minimize the distance *d* along the walk.



16. Find the angle θ which will maximize the area of the isosceles trapezoid shown in the figure.



17. A plank is used to reach over a fence 8 feet high to support a wall that is 1 foot behind the fence. What is the length of the shortest plank that can be used? [Hint: Express the length of the plank in terms of the angle θ shown in the figure.]